Chapter 3

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Homework Chapter 3

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3-1 Symmetry

Point Symmetry:

Two distinct points P and P' are symmetric with respect to point M if and only if M is the midpoint of PP'. Point M is symmetric with respect to itself.

Another way of thinking about it is if the figure (graph) is rotated 180 degrees about the point, it is unchanged.

Typically the origin is the point of symmetry.

\[
f(x) = x^3
\]

\[
g(x) = \frac{1}{x}
\]
What can we conclude when we look at the table of values???

Example 1: Determine whether each graph is symmetric with respect to the origin.

a. \( f(x) = x^5 \)
1. Determine whether the graph is symmetric with respect to the origin.
   \[ y = x^6 \]
   Yes
   No

2. Determine whether the graph is symmetric with respect to the origin.
   \[ y = -3x^3 + 5x \]
   Yes
   No
Line Symmetry:

We've seen this before in Algebra 2. Last year we looked at vertical and horizontal parabolas with their Axis of Symmetry.

Two distinct points P and P' are symmetric with respect to a line \( l \) if \( l \) is the perpendicular bisector of PP'.

Common lines of symmetry include: x-axis, y-axis, y=x, and y=-x.
Symmetry Definitions

<table>
<thead>
<tr>
<th>Symmetry Type</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-axis symmetry</td>
<td>(a, -b) ∈ S iff (a, b) ∈ S</td>
</tr>
<tr>
<td>y-axis symmetry</td>
<td>(-a, b) ∈ S iff (a, b) ∈ S</td>
</tr>
<tr>
<td>y = x symmetry</td>
<td>(b, a) ∈ S iff (a, b) ∈ S</td>
</tr>
<tr>
<td>y = -x symmetry</td>
<td>(-b, -a) ∈ S iff (a, b) ∈ S</td>
</tr>
</tbody>
</table>

Plan: Substitute first and second ordered pairs into function and see if they are equivalent expressions.

Example 2: Determine whether the graph of $xy = -2$ is symmetric with respect to the x-axis, y-axis, y=x, y=-x or none of these.

x-axis

y-axis

y=x

y=-x
3. Determine whether the graph is symmetric with respect to x-axis, y-axis, y=x, y=-x, or none.

\[ x^2 + y = 3 \]

Example 3: Determine whether the graph of \(|y| = 2 - |2x|\) is symmetric with respect to the x-axis, the y-axis, both or neither. Use that information to graph the relation.
Even Functions:

Odd Functions:

3-2 Families of Graphs

Let's look closer at

\[ y = \sqrt{x} \]
Now let's move it around!

$y = -\sqrt{x}$

$y = \sqrt{x} + 2$
\[ y = \sqrt{x - 4} \]

\[ y = \sqrt{x + 3} - 1 \]
Example 1: Graph the parent function and the given function.

\[ g(x) = \frac{4}{x} \]

\[ h(x) = -0.25x^{-1} + 3 \]
1. What is the parent function for this graph? (answer must be $y =$...)

2. What translations were done?

$$g(x) = (x + 1)^2 - 2$$

A. Left 1 unit, up 2 units
B. Right 2 units, down 1 unit
C. Left 1 unit, down 2 units
D. Left 2 units, down 1 unit
E. None of the above
What happens to the graph when we take absolute values?

\[ |g(x)| = |x + 1|^2 - 2 |\]

\[ g(|x|) = (|x| + 1)^2 - 2 \]

3  Regarding translations ...

   A  I am ready to practice what I know.
   B  I want a few more examples.
   C  I have a couple questions.
1. Determine which points are solutions for the inequality. (MAP)

\[ y \geq 2x^3 + 7 \]

A \((-2,5)\)
B \((3,-1)\)
C \((-4,2)\)
D \((-1,-1)\)
Example 1: Graph

\[ y \leq (x + 1)^3 \]

Example 2: Graph

\[ y > -|x - 4| + 2 \]
Example 3: Solve \[ 3 + |x - 4| > 8 \]

Example 4: Solve \[ |3x - 4| \leq x \]
2 Solve \[ |5x + 8| > 10 \]
A  \( x < \frac{2}{5} \)
B  \( x > \frac{2}{5} \)
C  \( x < -\frac{18}{5} \)
D  \( x > -\frac{18}{5} \)
E  None of the above

3 Solve \[ 3|2x - 1| < -9 \]
A  \( x < -1 \)
B  \( x > -1 \)
C  \( x < 2 \)
D  \( x > 2 \)
E  None of the above
4 Regarding inequalities ...

A I am ready to practice what I know.
B I want a few more examples.
C I have a couple questions.

3-4 Inverse Functions and Relations

Inverse Relations:

Horizontal Line Test:
Example 1: Graph $f(x) = x^2$ and its inverse.

Example 2: Consider $f(x) = x^2 - 4$

a. Is the inverse of $f(x)$ a function?

b. Find $f^{-1}(x)$

c. Graph both.
1. Find the inverse of the function.

\[ f(x) = (x + 2)^2 + 6 \]

2. Is the inverse a function from #1?

Yes

No
Example 3: Graph

\[ y = 2 + \sqrt[3]{x - 7} \]

Parent graph?

\[ y = x^3 \]

Inverse Functions:

Two functions, \( f \) and \( f^{-1} \), are inverse functions iff

\[
\begin{align*}
[f \circ f^{-1}](x) &= [f^{-1} \circ f](x) = x
\end{align*}
\]

Example 4: Given \( f(x) = 3x^2 + 7 \), find \( f^{-1}(x) \), and verify they are inverses using the above definition.
3 Verify that they are inverses.

\[ f(x) = \frac{1}{4} x + \frac{9}{4} \quad f^{-1}(x) = 4x - 9 \]

Yes
No

4 Regarding inverses ...

A I am ready to practice what I know.
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C I have a couple questions.
3-5 Continuity and End Behavior

Continuous Function:
a smooth curve; you can trace the curve with your pencil
without lifting the point

Discontinuous Functions:

Infinite Discontinuity:
goes to infinity at a certain
x-value.

Jump Discontinuity:
the graph stops at a given
value of the domain and then
begins again at a different
range value for the same
value of the domain.

Point Discontinuity:
where a value in the domain
for which the function is
undefined
Continuity Test:
A function is continuous at \( x = c \) if it satisfies 3 things:
1. The function is defined at \( c \); \( f(c) \) exists.
2. The function approaches the same \( y \)-value on the left and right of \( c \)
3. The \( y \)-value that the function approaches from each side is \( f(c) \)

Example 1: Determine whether each function is continuous at the given \( x \)-value.

a. \( f(x) = 3x^2 + x - 7 \) at \( x = 1 \)

b. \( f(x) = \frac{x^2 - 4}{x + 2} \) at \( x = -2 \)

c. \( f(x) = \begin{cases} 
  x^2 & \text{if } x \geq -2 \\
  \frac{1}{x^2 - 4} & \text{if } x < -2
\end{cases} \) at \( x = -2 \)

1. Determine if the function is continuous at \( x = -2 \).

\[
y = \frac{x - 5}{x + 3}
\]

Yes
No
Continuity on an Interval:
A function $f(x)$ is continuous on an interval iff it is continuous at each number $x$ in the interval.

2 Is the greatest integer function continuous?

$f(x) = \lceil x \rceil$

Yes
No
End Behavior
What happens as \( x \to +\infty \) and \( x \to -\infty \)

Example 2: Describe the end behavior for the functions

\[ f(x) = 5x^3 \quad g(x) = -5x^3 + 4x^2 - 2x + 4 \]
End Behavior for Polynomial Functions

\[ p(x) = a_n x^n + a_{n-1} x^{n-1} \ldots a_1 x + a_0 \]

Increasing, Decreasing, and Constant Functions

**Increasing:**
if for every values \( a \) and \( b \) on an interval, \( f(a) < f(b) \) when \( a < b \)

**Decreasing:**
if for every values \( a \) and \( b \) on an interval, \( f(a) > f(b) \) when \( a > b \)

**Constant:**
if for every values \( a \) and \( b \) on an interval, \( f(a) = f(b) \) when \( a < b \)
Example 3: Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.

a. \( f(x) = 3 - (x-5)^2 \)

b. \( f(x) = \frac{1}{2} |x+3| - 5 \)

3. When is the function increasing?

\( f(x) = (x + 3)^2 - 4 \)
4 When is the function decreasing?

\[ f(x) = (x + 3)^2 - 4 \]

5 Regarding Continuity and End Behavior ... 

A I am ready to practice what I know.

B I want a few more examples.

C I have a couple questions.
3-6 Critical Points and Extrema

Critical Points:

Point of Inflection (IP):

Absolute Maximum:

Absolute Minimum:

Extremum:
Relative Extrema:

Relative maximum:

Relative Minimum:

Example 1: Locate the extrema for the graph. Name and classify the extrema.

\[ y = |x^2 - 2| \]
Example 2: The function $f(x) = 3x^4 - 4x^3$ has critical points at $x=0$ and $x=1$. Determine whether each of these critical points is the location of a maximum, a minimum or a point of inflection.

Let $h=0.1$
1. Determine the type of critical point for $x = -2.5$.
   \[ y = x^2 + 5x - 6 \]
   A. Maximum
   B. Minimum
   C. Point of Inflection

2. Determine the type of critical point for $x = 0$.
   \[ y = 2x^3 - x^5 \]
   A. Maximum
   B. Minimum
   C. Point of Inflection
3 Regarding ...

A I am ready to practice what I know.
B I want a few more examples.
C I have a couple questions.

3-7 Rational Functions

\[ f(x) = \frac{g(x)}{h(x)} \]

Parent Function:

\[ f(x) = \frac{1}{x} \]
Vertical Asymptotes:

Horizontal Asymptotes:

Example 1: Determine the asymptotes for the graph of

\[ f(x) = \frac{3x - 1}{x - 2} \]

Vertical Asymptote:

Horizontal Asymptote:
2 possible methods
1. Determine the vertical asymptote.

\[ f(x) = \frac{x}{x - 5} \]

2. Determine the horizontal asymptote.

\[ f(x) = \frac{x}{x - 5} \]
Example 2: Using the parent graph from graph each function and describe the translation.

\[ g(x) = \frac{1}{x - 1} \]

\[ h(x) = -\frac{2}{x} \]
MA Chapter 3 Notes

\[ k(x) = \frac{7}{x + 5} \]

\[ m(x) = \frac{1}{x - 3} + 2 \]
Slant Asymptotes:
degree of the numerator is exactly one greater than denominator.

\[ f(x) = \frac{x^3 + 1}{x^2} \]

Example 3: Determine the slant asymptote.

\[ f(x) = \frac{4x^2 + 6x - 37}{x + 4} \]
Where do Point Discontinuities come from?

\[ f(x) = \frac{(x + 2)(x - 3)}{x - 3} \]

Example 4: Graph

\[ f(x) = \frac{(x + 2)(x - 3)}{x(x - 4)^2(x + 2)} \]
3. Determine the point discontinuity. Ordered pair answer.

\[ y = \frac{x^2 + 3x + 2}{x + 2} \]

4. Write the equation for the graph shown. The parent graph is below.

\[ y = \frac{1}{x} \]
5 Determine the slant asymptote for the function.

\[ f(x) = \frac{3x^2 - 4x + 5}{x - 3} \]

6 Regarding asymptotes ...

A I am ready to practice what I know.
B I want a few more examples.
C I have a couple questions.
3-8 Direct, Inverse and Joint Variation

Direct Variation

\[ y = kx^n \]

Constant of Variation

Example 1: Suppose \( y \) varies directly as \( x \) and \( y = 45 \) when \( x = 2.5 \).

a. Find the constant of variation and write the equation.
b. Use the equation to find the value of \( y \) when \( x = 4 \).

When an object such as a car is accelerating, twice the distance \( d \) it travels varies directly with the square of the time \( t \) elapsed. One car accelerating for 4 minutes travels 1440 feet.

a. Write an equation of direct variation relating distance to time. Graph the equation.
b. Use the equation to find the distance travelled by the car in 8 minutes.
Direct Variation problems may also be solved using proportions.

\[ y_1 = kx_1^n \quad \text{and} \quad y_2 = kx_2^n \]

1. If y varies directly as the cube of x and y=-67.5 when x=3, find x when y=-540. (NOA)
Inverse Variation

\[ x^n y = k \quad \text{or} \quad y = \frac{k}{x^n} \]

When you travel to higher elevation above the Earth’s surface the air temperature decreases.

Proportions can be used as well to solve Inverse Variation problems.

2. If \( y \) varies inversely as \( x \) and \( y=14 \) when \( x=3 \), find \( x \) when \( y=30 \). (NOA)
3 What is the constant of variation?

\[ \frac{x^4}{y} = 7 \]

Joint Variation
3 Variables!

\[ y = kx^n z^n \]

Example 3: The volume V of a cone varies jointly as the height h and the square of the radius r. Find the equation for the volume with height 6 cm and diameter of 10 cm that has a volume of $50\pi$ cubic centimeters.
4 Regarding variations...
   A I am ready to practice what I know.
   B I want a few more examples.
   C I have a couple questions.
Graph each function.

1. \( y = 0 \)

2. \( y = x \)
3. $y = x^2$

4. $y = x^3$
5. \( y = |x| \)

6. \( y = [x] \)
7. \( y = \sqrt{x} \)

8. \( y = \frac{1}{x} \)