

sample size was 100? Explain your reasoning, including a sketch of the sampling distribution.

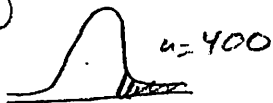
The probability would drop significantly. The larger  $n$  yields a  $\sigma_{\bar{x}}$  half of the standard error w/  $n=100$ . This tightens the curve about the mean, lowering the area past .25.



(h) Calculate the probability asked for in (g). Was your conjecture correct? Explain.  
 $\sigma_{\bar{x}} = \frac{.022}{2}$   
 $P(\bar{x} \geq .25) = 1 - .587 = .413$  For a 16% likelihood the average proportion of smokers in a group of 400 is more than .25.

(i) Repeat (g) and (h) for a sample of size 1600.

The probability would fall again because the  $\sigma_{\bar{x}}$  of the 400 sample size is now divided by 20. This yields a tighter curve with less data past .25.



$$\sigma_{\bar{x}} = 2.100/20 = .105$$

$$P(\bar{x} \geq .25) = \frac{2.0 \times 10^{-89}}{1600} \approx 0\%$$

For a 0% likelihood the avg of 1600 is .25

The Center for Disease Control and Prevention further reported that 14.2% of Utah residents smoked regularly in 1998. Treat these as the parameter values for these states' populations.

(j) Suppose that you take a random sample of 100 residents from Utah and find the proportion of smokers in the sample. Sketch the sampling distribution of the sample proportion.

14.2 residents



(k) Use the CLT to calculate the probability that this sample proportion would exceed .25.

$$P(\bar{x} \geq .25) = 9.886 \times 10^{-4}$$

For .01% likelihood that there would be .25 smokers in 100 random Utah residents.

(l) Would the probability in (k) increase, decrease or stay the same if the sample size were larger? Explain your reasoning. The probability would decrease; a larger sample would tighten the gap around the mean and reduce the area above .25.

(m) If a random sample of 100 residents from an unknown state reveals 25 smokers, would you have strong reason to doubt that the state was Utah? Explain your reasoning.

Yes. Since there is only .01% chance that a 100-person sample in Utah would yield .25 smokers, the state would not be Utah. Additionally, as the national proportion was 22.9/100, some states would have to be closer to 25/100 than Utah, much more likely candidates.

- (h) Which of your above answers would be affected if the distribution of the weights of the bags was not normal but was rather skewed?

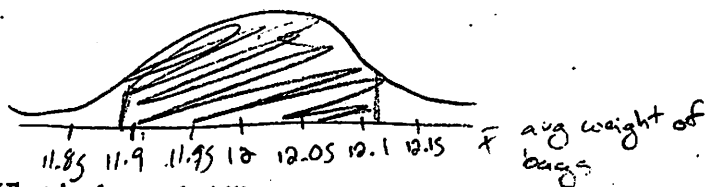
The answers to part (a) and (c) because they are normal curve calculations. The reason that the rest would be fine is that the CLT says that with  $n \geq 30$ , normal curves can be used for mean distributions.

- (i) What is the probability that the sample mean weight from a sample of 50 randomly selected bags would be between 11.89 and 12.11 ounces?

$$n = 50 \quad \sigma_{\bar{x}} = .0566$$

$$P(11.89 \leq \bar{x} \leq 12.11) = .9485$$

$\exists$  95% likelihood that the average weight

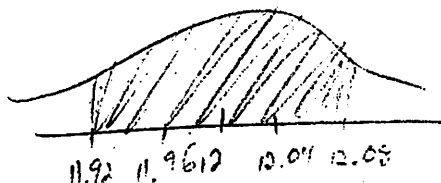


- (j) What is the probability that the sample mean weight from a sample of 100 randomly selected bags would be between 11.92 and 12.08 ounces?

$$\sigma_{\bar{x}} = .04$$

$$P(11.92 \leq \bar{x} \leq 12.08) = .9541$$

$\exists$  95% likelihood that the average weight of 100 bags will be b/w 11.92 and 12.08



- (k) Find a value  $k$  such that the probability of the sample mean weight of 1000 randomly selected bags being between  $12-k$  and  $12+k$  is roughly 0.95.

$$\sigma_{\bar{x}} = .012689$$

$$P(12-k \leq \bar{x} \leq 12+k) = .95$$

$$k = 1.96(\sigma_{\bar{x}})$$

$$k = .02479$$

- (l) What is the smallest sample size for which the probability that the sample mean falls between 11.95 and 12.05 is at least .95?

$$P(11.95 \leq \bar{x} \leq 12.05) = .95 \quad n = ?$$

$$.05 = 1.96(\sigma_{\bar{x}})$$

$$\sigma_{\bar{x}} = .02551$$

$$\sigma_{\bar{x}} = \frac{.01}{\sqrt{n}}$$

$$\frac{.01}{.02551} = \sqrt{n}$$

$$n = 246 \text{ bags}$$

Smoking Rates:

The Center for Disease Control and Prevention reported that 22.9% of American adults smoked regularly in 1998. Treat this as the parameter value for the population of American adults.

- (a) What symbol would represent this proportion of .229?

$p, \pi$

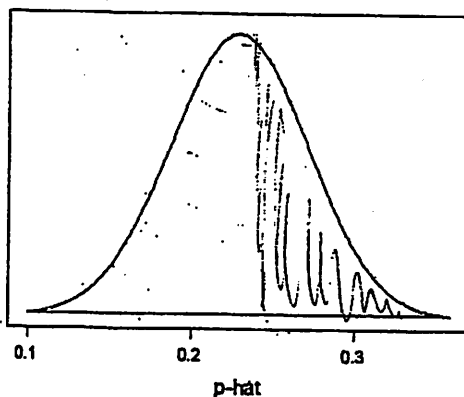
- (b) Suppose that you take a random sample of 100 American adults. Will the sample proportion of smokers equal exactly .229? Explain.

no; just as flipping a coin 100 times will not necessarily yield 50 tails/heads. Also, it is impossible for 22.9 of 100 people to have smoked, .9 smoker is impossible.

- (c) What does the CLT say about how this sample proportion of smokers would vary from sample to sample? [Be sure to comment on shape, center, and spread.]

If you took many 100-person samples, the proportion of smokers will be normally distributed around .229 because  $np \geq 10$  &  $nq \geq 10$ . The standard deviation would be 4.201 smokers.

- (d) On the following sketch of this sampling distribution, shade the area corresponding to a sample proportion exceeding .25. Based on this area, make a guess for the probability that the sample proportion of smokers would exceed .25.



40%

- (e) Using the CLT result, find the z-score corresponding to a sample proportion of smokers equal to .25.

$$\sigma_{\hat{p}} = 4.201 \quad \mu_{\hat{p}} = 22.9 \quad n = 100$$

$$\frac{25 - 22.9}{4.201} = z = 1.4999$$

- (f) Use this z-score and the table of normal probabilities to calculate the probability that the sample proportion of smokers would exceed .25.

$$P(z \geq .499) = .3086$$

Ex 31% chance that the proportion of smokers exceeds .25.

- (g) Suppose that you take a random sample of 400 American adults. How would you expect the probability of the sample proportion of smokers exceeding .25 to change from when the

43/3 e.c. Alouir Cohen  
per 6

This result is so important that it is called the Central Limit Theorem of statistics:

Suppose that a simple random sample of size  $n$  is taken from a large population in which the variable of interest has mean  $\mu$  and standard deviation  $\sigma$ . Then, provided that  $n$  is large (at least 30 as a rule-of-thumb), the sampling distribution of the sample mean  $\bar{X}$  is approximately normal with mean  $\mu$  and standard deviation  $\sigma/\sqrt{n}$ . The approximation holds with large sample sizes regardless of the shape of the population distribution. The accuracy of the approximation increases as the sample size increases. For populations that are themselves normally distributed, the result holds not approximately but exactly.

Notes:

- The mean and standard deviation results always hold (see above).
- Since we can view the sample proportion  $\hat{p}$  as the average of 0's and 1's, we can apply this result to the sampling distribution of  $\hat{p}$ . Thus, the sampling distribution of

$\hat{p}$  is approximately normal with mean  $p$  and standard deviation  $SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$ .

This approximation improves as the sample size increases. Recall from the normal approximation to the binomial,  $n$  is considered large enough if  $np \geq 10$  and  $n(1-p) \geq 10$ . Some texts use 5 as the cut-off for this rule of thumb.

#### Potato Chips:

Suppose that the weights of bags of potato chips coming off an assembly line are normally distributed with mean 12 ounces and standard deviation 0.4 ounces.

- (a) What is the probability that one randomly selected bag weighs less than 11.9 ounces?

$$P(X \leq 11.9) = 0.4013$$

For 40% likelihood that a random bag weighs under 11.9 oz.



- (b) If you took a random sample of ten bags, would you expect the probability of their sample mean weight being less than 11.9 ounces to be greater or less than the probability found in a)? Explain, without performing the calculation.

I would expect the probability to be less. A larger sample decreases the standard error ( $\sigma_{\bar{x}}$ ) which tightens the curve around the mean. This means that less info will fall far from the mean.

- (c) Calculate the probability asked about in the previous question.  $n=10$   $\mu_x = 12 \text{ oz}$   $\sigma_x = .1265$

$$P(\bar{x} \leq 11.9) = .2146$$

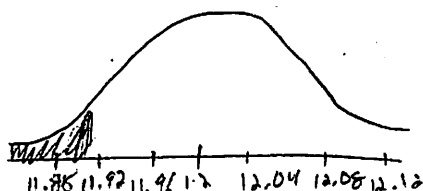
For a 21.5% likelihood that the avg. weight of 10 random bags is under 11.9 oz.



- (d) Repeat this analysis, for a sample of 100 randomly selected bags.  $n=100$   $\sigma_{\bar{x}} = .04$

$$P(\bar{x} \leq 11.9) = .006211$$

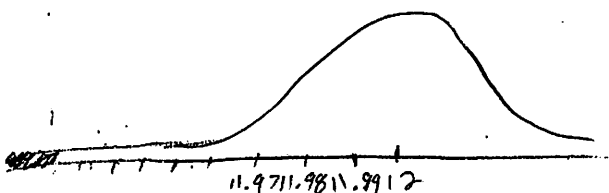
For .6% chance the avg weight of 100 random bags is under 11.9 oz.



- (e) Repeat this analysis for a random sample of 1000 bags.  $\sigma_{\bar{x}} = .0126$   $n=1000$

$$P(\bar{x} \leq 11.9) = 0$$

For 0% likelihood the avg weight of 1000 bags is less than 11.9 oz.



- (f) What is the smallest sample size for which the probability of the sample mean being less than 11.9 ounces is less than .01?

$$P(\bar{x} \leq 11.9) < .01 \quad n=?$$

$$\sigma_{\bar{x}} = \frac{.4}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = .0429852$$

$$P(\bar{x} \leq 11.9) = .009999$$

$$\frac{.4}{.0429852} = \sqrt{n}$$

$$\sqrt{n} = 9.3055$$

$$n = 86.5 \rightarrow 87 \text{ bags}$$

- (g) If you were told that a consumer group had weighed randomly selected bags and found a sample mean weight of 11.9 ounces, would you doubt the claim that the true mean weight of all of the potato chip bags is 12 ounces? On what unspecified information does your answer depend? Explain.

I would not necessarily doubt the mean of 12 oz.

It depends on the sample size. If the sample was

5, I would be fine with the data, but if  $n=100$ ,

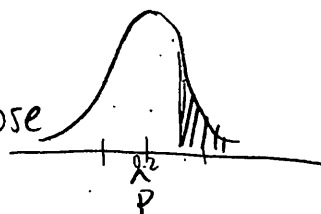
I would doubt the average of 12 oz.

guess  
check

sample size was 100? Explain your reasoning, including a sketch of the sampling distribution.

$$SD(\hat{p}) = \sqrt{\frac{0.229(1-0.229)}{100}} = 0.021$$

This concentrates the data values close to the mean so the area above 0.25 is smaller.



(h) Calculate the probability asked for in (g). Was your conjecture correct? Explain.

$$\hat{P}(>0.25) = \text{normalcdf}(0.25, 1000, 0.229, 0.021) = 0.1587$$

Ex a 15.87% chance that the sample proportion is above 0.25

(i) Repeat (g) and (h) for a sample of size 1600. since sample size is even larger, there is less spread so the probability of a sample proportion above 0.25 will decrease even more.

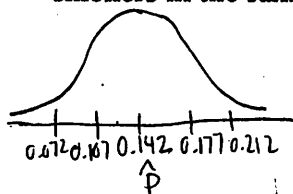
$$SD(\hat{p}) = \sqrt{\frac{0.229(1-0.229)}{1600}} = 0.01050$$

$$\hat{P}(>0.25) = \text{normalcdf}(0.25, 1000, 0.229, 0.0105) = 0.02275$$

Ex a 2.275% chance that the sample proportion is above 0.25

The Center for Disease Control and Prevention further reported that 14.2% of Utah residents smoked regularly in 1998. Treat these as the parameter values for these states' populations.

(j) Suppose that you take a random sample of 100 residents from Utah and find the proportion of smokers in the sample. Sketch the sampling distribution of the sample proportion.



$$SD(\hat{p}) = \sqrt{\frac{0.142(1-0.142)}{100}} = 0.0349$$

(k) Use the CLT to calculate the probability that this sample proportion would exceed .25.

$$\hat{P}(>0.25) = \text{normalcdf}(0.25, 1000, 0.142, 0.0349) = 0.001$$

Ex a 0.1% chance that the sample proportion is larger than 0.25.

(l) Would the probability in (k) increase, decrease or stay the same if the sample size were larger? Explain your reasoning.

Decrease, because the data values will be even less spread out and probability above 0.25 will go down.

(m) If a random sample of 100 residents from an unknown state reveals 25 smokers, would you have strong reason to doubt that the state was Utah? Explain your reasoning.

$$\frac{25}{100} = 0.25$$

Yes, because we saw that in a 100 sample, the sample proportion is larger than 0.25 only 0.1% of the time.

Smoking Rates:

The Center for Disease Control and Prevention reported that 22.9% of American adults smoked regularly in 1998. Treat this as the parameter value for the population of American adults.

- (a) What symbol would represent this proportion of .229?

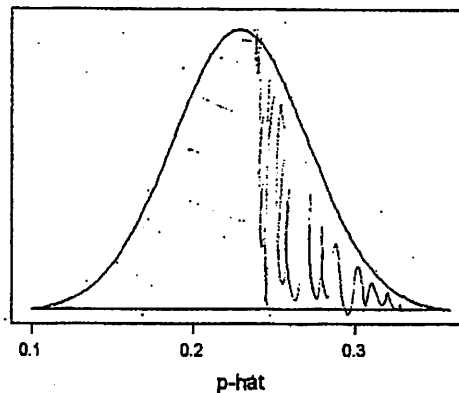
$p$

- (b) Suppose that you take a random sample of 100 American adults. Will the sample proportion of smokers equal exactly .229? Explain.  
No, because it depends on sample size. As sample size increases, it should get closer and closer to 0.229.

- (c) What does the CLT say about how this sample proportion of smokers would vary from sample to sample? [Be sure to comment on shape, center, and spread.]

It's approximately normal, and the  $p = 0.229$ , and  
 $SD = \sqrt{\frac{p(1-p)}{n}}$

- (d) On the following sketch of this sampling distribution, shade the area corresponding to a sample proportion exceeding .25. Based on this area, make a guess for the probability that the sample proportion of smokers would exceed .25.



About 40%.

- (e) Using the CLT result, find the z-score corresponding to a sample proportion of smokers equal to .25.

$$SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.229(0.771)}{100}} = 0.042$$

$$z\text{-score} = \frac{0.25 - 0.229}{0.042} = 0.5$$

- (f) Use this z-score and the table of normal probabilities to calculate the probability that the sample proportion of smokers would exceed .25.

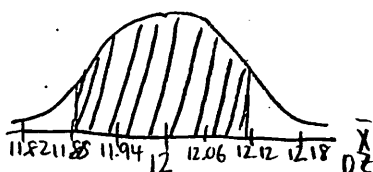
$$P(\hat{p} > 0.25) = 1 - 0.6915 = 0.3085$$

- (g) Suppose that you take a random sample of 400 American adults. How would you expect the probability of the sample proportion of smokers exceeding .25 to change from when the

- (h) Which of your above answers to would be affected if the distribution of the weights of the bags was not normal but was rather skewed?

b because the sample size (10) is too small

- (i) What is the probability that the sample mean weight from a sample of 50 randomly selected bags would be between 11.89 and 12.11 ounces?



$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{50}} = 0.05657$$

$$P(11.89 < \bar{x} < 12.11) = \text{normalcdf}(11.89, 12.11, 12, 0.05657) = 0.9482$$

∴ a 94.82% chance that the sample mean weight is between 11.89 and 12.11 oz.

- (j) What is the probability that the sample mean weight from a sample of 100 randomly selected bags would be between 11.92 and 12.08 ounces?

$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{100}} = 0.04$$



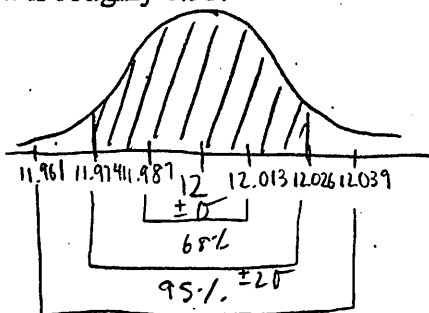
$$P(11.92 < \bar{x} < 12.08) = \text{normalcdf}(11.92, 12.08, 12, 0.04) = 0.9545$$

∴ a 95.45% chance that on average, the sample mean weight is between 11.92 and 12.08 oz.

- (k) Find a value  $k$  such that the probability of the sample mean weight of 1000 randomly selected bags being between  $12-k$  and  $12+k$  is roughly 0.95.

$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{1000}} = 0.01265$$



$$k = 2\sigma_{\bar{x}} = 2(0.01265) = 0.0253$$

∴ a 95% chance that the sample mean weight is between 11.9747 and 12.0253.

- (l) What is the smallest sample size for which the probability that the sample mean falls between 11.95 and 12.05 is at least .95?

95% of sample mean is within  $2\sigma_{\bar{x}}$  from mean.

$$2\sigma_{\bar{x}} = 0.05$$

$$\sigma_{\bar{x}} = \frac{0.05}{2} = 0.025$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$0.025 = \frac{0.4}{\sqrt{n}}$$

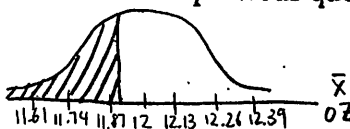
$n = 256$  is smallest sample size



- (c) Calculate the probability asked about in the previous question.

$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{10}} = 0.1265$$



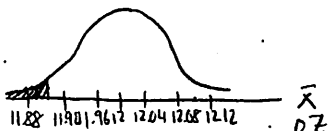
$$P(\bar{x} < 11.9) = \text{normalcdf}(0, 11.9, 12, 0.1265)$$

= 0.2146  $\exists$  a 21.46% chance that the sample mean weight is less than 11.9 oz.

- (d) Repeat this analysis, for a sample of 100 randomly selected bags.

$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{100}} = 0.04$$



$$P(\bar{x} < 11.9) = \text{normalcdf}(0, 11.9, 12, 0.04)$$

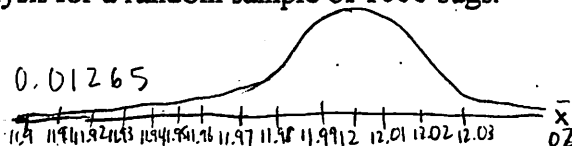
$$= 0.0062$$

$\exists$  a 0.62% chance that the sample mean weight is less than 11.9 oz.

- (e) Repeat this analysis for a random sample of 1000 bags.

$$\mu_{\bar{x}} = 12$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{1000}} = 0.01265$$



$$P(\bar{x} < 11.9) = \text{normalcdf}(0, 11.9, 12, 0.01265)$$

$\exists$  essentially a 0% chance that the sample mean weight is less than 11.9 oz.

- (f) What is the smallest sample size for which the probability of the sample mean being less than 11.9 ounces is less than .01?

$$P(\bar{x} < 11.9) < 0.01$$

$$\text{InvNorm}(0.01, 12, 0.04)$$

$$\begin{aligned} \hookrightarrow \sigma_{\bar{x}} &= \frac{\sigma}{\sqrt{n}} \\ 0.04 &= \frac{0.4}{\sqrt{n}} \\ n &= 100 \end{aligned}$$

- (g) If you were told that a consumer group had weighed randomly selected bags and found a sample mean weight of 11.9 ounces, would you doubt the claim that the true mean weight of all of the potato chip bags is 12 ounces? On what unspecified information does your answer depend? Explain.

I would not doubt it because the sample mean weight depends on sample size. The sample mean will get closer to 12 as the sample size gets larger.

This result is so important that it is called the Central Limit Theorem of statistics:

Suppose that a simple random sample of size  $n$  is taken from a large population in which the variable of interest has mean  $\mu$  and standard deviation  $\sigma$ . Then, provided that  $n$  is large (at least 30 as a rule-of-thumb), the sampling distribution of the sample mean  $\bar{X}$  is approximately normal with mean  $\mu$  and standard deviation  $\sigma/\sqrt{n}$ . The approximation holds with large sample sizes regardless of the shape of the population distribution. The accuracy of the approximation increases as the sample size increases. For populations that are themselves normally distributed, the result holds not approximately but exactly.

Notes:

- The mean and standard deviation results always hold (see above).
- Since we can view the sample proportion  $\hat{p}$  as the average of 0's and 1's, we can apply this result to the sampling distribution of  $\hat{p}$ . Thus, the sampling distribution of

$\hat{p}$  is approximately normal with mean  $p$  and standard deviation  $SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$ .

This approximation improves as the sample size increases. Recall from the normal approximation to the binomial,  $n$  is considered large enough if  $np \geq 10$  and  $n(1-p) \geq 10$ . Some texts use 5 as the cut-off for this rule of thumb.

### Potato Chips:

Suppose that the weights of bags of potato chips coming off an assembly line are normally distributed with mean 12 ounces and standard deviation 0.4 ounces.

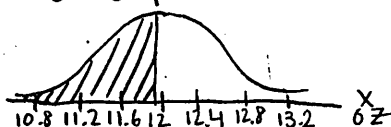
- (a) What is the probability that one randomly selected bag weighs less than 11.9 ounces?

$$\mu = 12$$

$$\sigma = 0.4$$

$$P(X < 11.9) = \text{normcdf}(0, 11.9, 12, 0.4)$$

= 0.4013 or a 40.13% chance that the bag weighs less than 11.9 oz.



- (b) If you took a random sample of ten bags, would you expect the probability of their sample mean weight being less than 11.9 ounces to be greater or less than the probability found in a)? Explain, without performing the calculation.

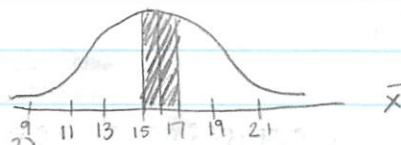
Since the  $\sigma$  will be smaller, the data values will be more concentrated and condensed around the  $\mu$  and thus, a smaller probability would be less than 11.9 oz because there is less spread and deviation from the average.

# 8.2

1a)  $\mu_{\bar{x}} = 15$

$\sigma_{\bar{x}} = \frac{14}{\sqrt{49}} = 2$

$P(15 \leq \bar{x} \leq 17) = \text{normalcdf}(15, 17, 15, 2)$   
 $= 0.3413$

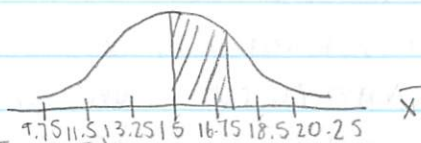


$\exists$  a 34.13% chance that on average, the mean is between 15 and 17.

b)  $\mu_{\bar{x}} = 15$

$\sigma_{\bar{x}} = \frac{14}{\sqrt{64}} = 1.75$

$P(15 \leq \bar{x} \leq 17) = \text{normalcdf}(15, 17, 15, 1.75)$   
 $= 0.3735$



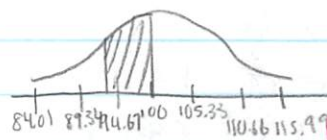
$\exists$  a 37.35% chance that on average, the mean is between 15 and 17.

c) Because  $n$  is bigger in (b), which causes a lower  $\sigma_{\bar{x}}$ , and concentrates the data so there's a higher prob

2a)  $\mu_{\bar{x}} = 100$

$\sigma_{\bar{x}} = \frac{48}{\sqrt{81}} = 5.33$

$P(92 \leq \bar{x} \leq 100) = \text{normalcdf}(92, 100, 100, 5.33)$   
 $= 0.4333$



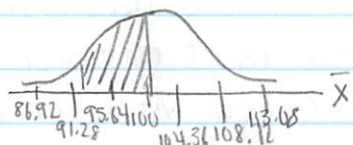
$\bar{x} = \text{avg of ?}$

$\exists$  a 43.33% chance that on average between 92 and 100 are successful.

b)  $\mu_{\bar{x}} = 100$

$\sigma_{\bar{x}} = \frac{48}{\sqrt{121}} = 4.36$

$P(92 \leq \bar{x} \leq 100) = \text{normalcdf}(92, 100, 100, 4.36)$   
 $= 0.4667$



$\exists$  a 46.67% chance that on average between 92 and 100 are successful.

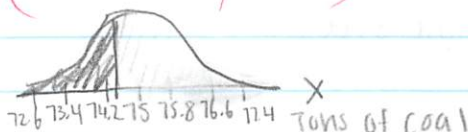
c) The prob is higher in (b) because of the lower  $\sigma_{\bar{x}}$  and more concentrated data. The sample size is larger, which decreases  $\sigma_{\bar{x}}$  and increases probability. — (in center/mid)

5a)  $\mu = 75, \sigma = 0.8$

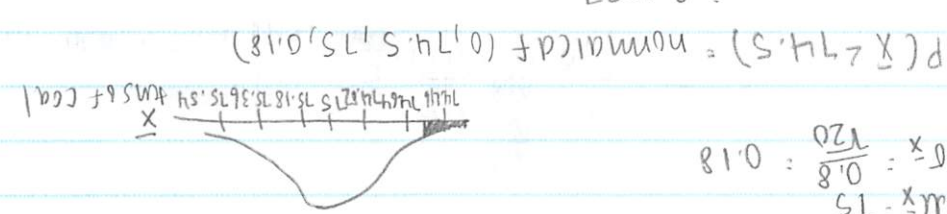
$P(X < 74.5) = 0.2660$

$\exists$  a 26.60% chance that

a car will have less than 74.5 tons of coal.



5b)  $\mu_X = 75$   
 $\sigma_X = \frac{120}{0.8} = 0.18$

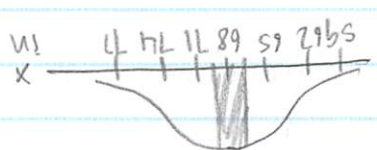


$P(X < 74.5) = \text{normalcdf}(0, 74.5, 75, 0.18)$   
 $= 0.0027$

Ex a 0.27% chance that on average, the mean load of 20 cars is less than 74.5 tons of coal.

c) If it was just one car, no because 26.60% of cars have less than 74.5 tons. With 20 cars, however, the average below 74.5 tons would be suspicious since that happens less than a percent of the time.

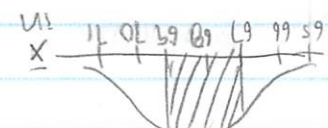
6a)  $\mu = 68, \sigma = 3$   
 $P(67 < X < 69) = \text{normalcdf}(67, 69, 68, 3)$   
 $= 0.2611$



Ex a 26.11% chance that a man at random is between 67 and 69 inches tall.

b)  $\mu_X = 68$   
 $\sigma_X = \frac{19}{3} = 1$

$P(67 < X < 69) = \text{normalcdf}(67, 69, 68, 1)$   
 $= 0.6827$



c) the prob in (b) was higher because the  $\sigma$ , and this makes the data much more concentrated.

10a) The red with sample size 100, blue with sample size 49. A larger sample size means more concentrated data.

b) sample size 49 bc data is more spread out  
 c) No, the  $\mu$  are same in population and sample. only the distribution is affected, not the mean.

12a) The  $X$  distribution is approximately normal because of the large sample size.

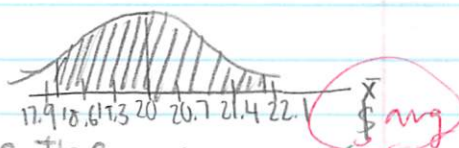
$\mu_X = 20$   
 $\sigma_X = \frac{1}{\sqrt{100}} = 0.1$

The assumption is that the sample size is large enough so with a random sample, the distribution will be approx. normal.



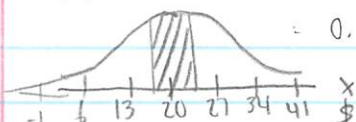
8.2

$$b) P(18 \leq \bar{x} < 22) = \text{normalcdf}(18, 22, 20, 0.47) = 0.9957$$



$\exists$  a 99.57% chance that the on average, the mean price is between 18 and 22.

$$c) P(18 < x < 22) = \text{normalcdf}(18, 22, 20, 7)$$



$$= 0.2249$$

$\exists$  a 22.49% chance that shoppers will spend between \$18 and \$22.

- d) The  $\bar{x}$  distribution has a smaller  $\sigma$  because of the large  $n$ . This makes the data much more concentrated and thus, predictable because a greater % of data values are clustered around the mean than in a regular  $x$  distribution.

5.8

1. The first part of the paper is devoted to a discussion of the general theory of the problem. It is shown that the problem is well-posed and that the solution exists and is unique. The second part of the paper is devoted to a discussion of the numerical solution of the problem. It is shown that the numerical solution is stable and that the error is of order  $O(h^2)$ . The third part of the paper is devoted to a discussion of the application of the theory to the problem of the motion of a particle in a magnetic field. It is shown that the theory predicts the existence of a stable orbit and that the numerical solution confirms this prediction.

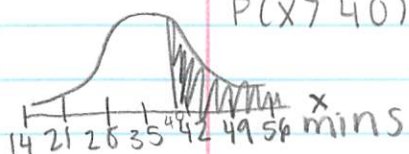
## ch 8 Rvw #1,3,4,5

- 1a) the  $\bar{x}$  distribution approaches a normal distribution.  
 b) the mean  $\mu$  of the  $x$  distribution  
 c)  $\frac{\sigma}{\sqrt{n}}$   
 d) they will both be approximately normal and have the same mean, but  $\sigma_{\bar{x}}$  will be  $\frac{\sigma}{\sqrt{50}}$  and  $\frac{\sigma}{\sqrt{100}}$

3a)  $\mu = 35$ ,  $\sigma = 7$

$P(X > 40) = \text{normalcdf}(40, 1000, 35, 7)$

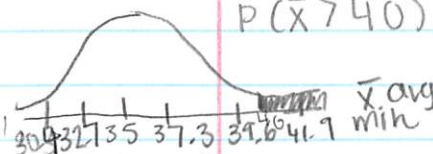
$= 0.2375$   $\Rightarrow$  a 23.75% chance that the interview



b)  $\mu_{\bar{x}} = 35$   $\sigma_{\bar{x}} = \frac{7}{\sqrt{4}} = 2.33$

$P(\bar{X} > 40) = \text{normalcdf}(40, 1000, 35, 2.33)$

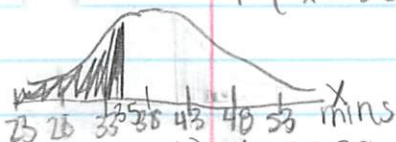
$= 0.01594$   $\Rightarrow$  a 1.594% chance that the average length of time will be 40 minutes or longer.



4a)  $\mu = 38$ ,  $\sigma = 5$

$P(X < 35) = \text{normalcdf}(0, 35, 38, 5)$

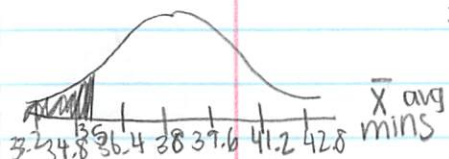
$= 0.2743$   $\Rightarrow$  a 27.43% chance that the time it takes to



b)  $\mu_{\bar{x}} = 38$   $\sigma_{\bar{x}} = \frac{5}{\sqrt{10}} = 1.58$

$P(\bar{X} < 35) = \text{normalcdf}(0, 35, 38, 1.58)$

$= 0.0288$   $\Rightarrow$  a 2.88% chance that the average time before it is effective for a 10 patients is 35 minutes or less.

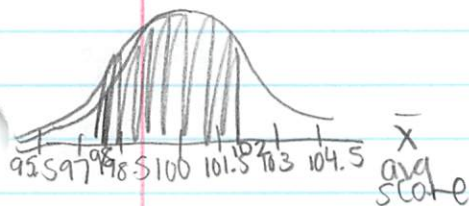


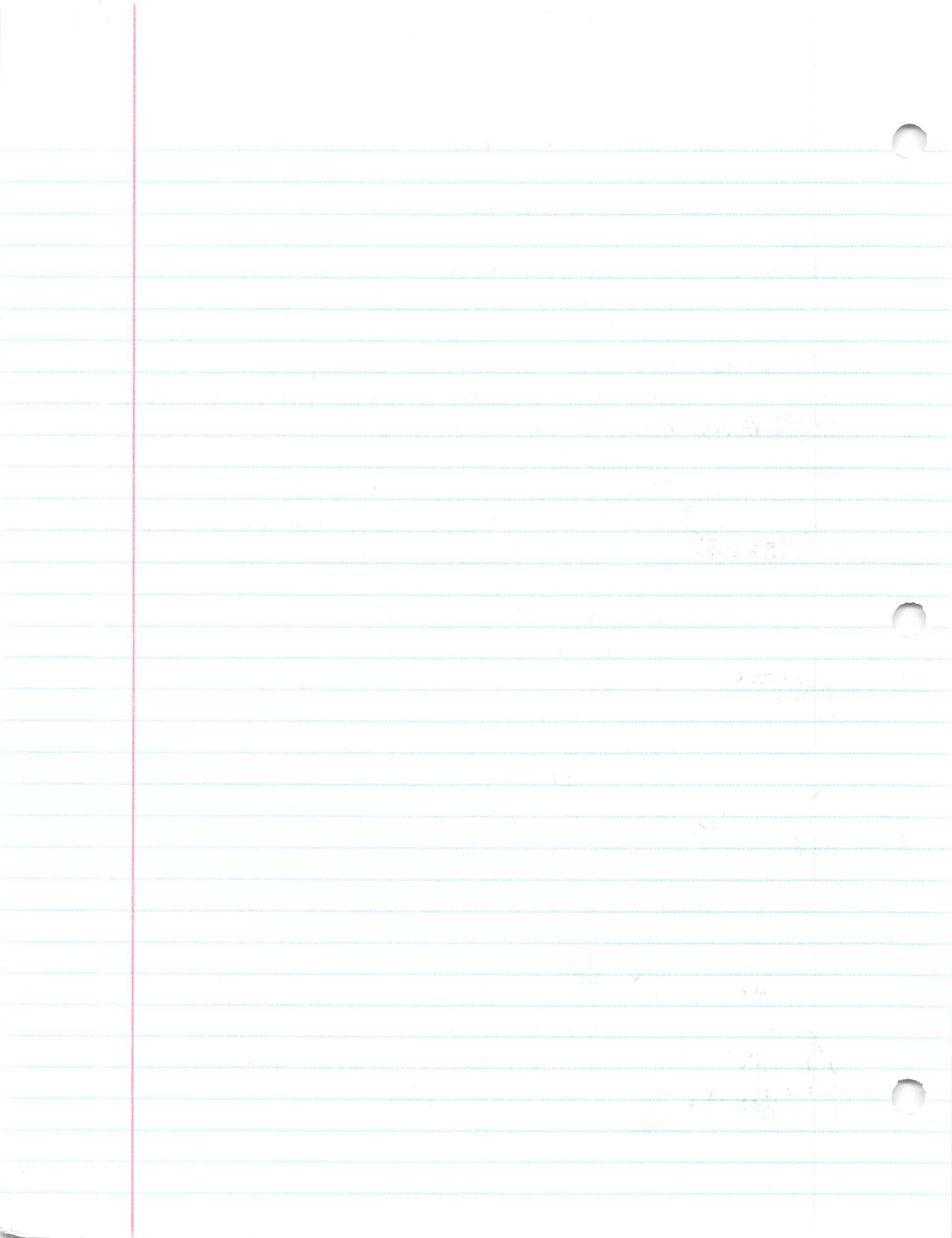
- c) the probability in part b is much smaller because the data is much more clustered around  $\mu$  and less will be on the tails.

5.  $\mu_{\bar{x}} = 100$   $\sigma_{\bar{x}} = \frac{15}{\sqrt{100}} = 1.5$

$P(98 < \bar{X} < 102) = \text{normalcdf}(98, 102, 100, 1.5)$

$= 0.8176$   $\Rightarrow$  an 81.76% chance that the sample mean will not differ from the population mean by more than 2 pts.







- c) Yes it seems somewhat old, especially given that he/she is a rookie player. 80% of the other players are between the ages of 27 and 28, so the 33 year old is relatively older.
- d)  $b) E = 2.58 \left( \frac{3.8}{\sqrt{40}} \right) \approx 1.55$

$$26.25 < \mu < 29.35$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of football players' ages 990 times. For this particular sample, we got an interval of  $26.25 < \mu < 29.35$ .

c) The player is old because 99% of the other players are between the ages of 26 and 29. Making the age of 33 an extreme outlier and very old compared to the rest of the players.

1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part, we consider the case of a single particle.

3. The third part is devoted to the case of a system of particles.

4. In the fourth part, we consider the case of a continuous medium.

5. The fifth part is devoted to the case of a system of continuous media.

6. In the sixth part, we consider the case of a single continuous medium.

7. The seventh part is devoted to the case of a system of continuous media.

8. In the eighth part, we consider the case of a single continuous medium.

9. The ninth part is devoted to the case of a system of continuous media.

10. In the tenth part, we consider the case of a single continuous medium.

11. The eleventh part is devoted to the case of a system of continuous media.

12. In the twelfth part, we consider the case of a single continuous medium.

13. The thirteenth part is devoted to the case of a system of continuous media.

14. In the fourteenth part, we consider the case of a single continuous medium.

15. The fifteenth part is devoted to the case of a system of continuous media.

16. In the sixteenth part, we consider the case of a single continuous medium.

17. The seventeenth part is devoted to the case of a system of continuous media.

18. In the eighteenth part, we consider the case of a single continuous medium.

19. The nineteenth part is devoted to the case of a system of continuous media.

20. In the twentieth part, we consider the case of a single continuous medium.

21. The twenty-first part is devoted to the case of a system of continuous media.

22. In the twenty-second part, we consider the case of a single continuous medium.

23. The twenty-third part is devoted to the case of a system of continuous media.

24. In the twenty-fourth part, we consider the case of a single continuous medium.

25. The twenty-fifth part is devoted to the case of a system of continuous media.

26. In the twenty-sixth part, we consider the case of a single continuous medium.

27. The twenty-seventh part is devoted to the case of a system of continuous media.

28. In the twenty-eighth part, we consider the case of a single continuous medium.

29. The twenty-ninth part is devoted to the case of a system of continuous media.

$$c) E = 1.645 \left( \frac{12.7}{\sqrt{35}} \right) = 3.53$$

$$142.97 < \mu < 150.03$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  <sup>of calories</sup> 900 times. For this particular sample, we got an interval of  $142.97 < \mu < 150.03$

$$d) E = 2.58 \left( \frac{12.7}{\sqrt{35}} \right) = 5.54$$

$$140.96 < \mu < 152.04$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  <sup>of calories</sup> 990 times. For this particular sample, we got an interval of  $140.96 < \mu < 152.04$

e) length in part (b) < length in part (c) < length in part (d)  
As the confidence level increases, so does the interval length.

$$10a) E = 1.96 \left( \frac{4.63}{\sqrt{30}} \right) = 1.66$$

$$14.05 < \mu < 17.37$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of base circumferences of aspen trees 950 times. For this particular sample, we got an interval of  $14.05 < \mu < 17.37$

$$b) E = 1.96 \left( \frac{4.61}{\sqrt{90}} \right) = 0.9524$$

$$14.63 < \mu < 16.53$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of circumferences of aspen trees 950 times. For this particular sample, we got an interval of  $14.63 < \mu < 16.53$

$$c) E = 1.96 \left( \frac{4.62}{\sqrt{300}} \right) = 0.5217$$

$15.07 < \mu < 16.11$  If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of circumferences of aspen trees 950 times. For this particular sample, we got an interval of  $15.07 < \mu < 16.11$



d) length of interval in part (a) = 3.32

part (b) = 1.90

part (c) = 1.04

As sample size increases and the  $\bar{x}$  approaches  $\mu$ , the length of the confidence level decreases.

11 a)  $E = 1.645 \left( \frac{0.7}{\sqrt{38}} \right) = 0.187$  Length of interval = 0.374

$$2.313 < \mu < 2.687$$

If we took a thousand samples of the <sup>same</sup> sample size, we expect to capture the pop mean  $\mu$  of time to go to sleep 900 times. For this particular sample, we got an interval of  $2.313 < \mu < 2.687$ .

b)  $E = 1.645 \left( \frac{4.8}{\sqrt{38}} \right) = 1.20$  Length of interval = 2.40

$$14 < \mu < 16.4$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of time to go to sleep for normal sleep college students 900 times. For this particular sample, we got an interval of  $14 < \mu < 16.4$ .

c)  $E = 1.645 \left( \frac{8.3}{\sqrt{38}} \right) = 2.21$  Length of interval = 4.42

$$23.49 < \mu < 27.91$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of time to go to sleep for college students w/ 12 hours of sleep 900 times. For this particular sample, we got an interval of  $23.49 < \mu < 27.91$ .

d) length of interval in part (a) < part (b) < part (c)

As the sample deviations got larger, the intervals got longer because in  $E = z_c \frac{s}{\sqrt{n}}$ , as  $s$  (the numerator) increases, so does  $E$ .

12 a)  $\bar{x} = 27.775$ ,  $s = 3.7994$

b)  $E = 1.28 \left( \frac{3.8}{\sqrt{40}} \right) = 0.7691$

$$27.03 < \mu < 28.57$$

If we took a thousand samples of the same sample size, we expect to capture the pop mean  $\mu$  of ages of pro football players 800 times. For this particular sample, we got an interval of  $27.03 < \mu < 28.57$ .