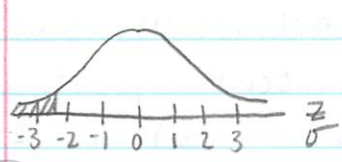
SUN

9

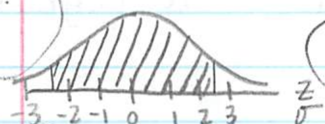
ch 7 RVW #6, 8, 10, 14, 18, 19

6. $z = -2.33$



8. $z = 2.58$

$-z = -2.58$



10a) $x = z\sigma + \mu$

$= (1.9)(35) + 270$

$= 336.5$

b) $x = z\sigma + \mu$

$= (-0.25)(35) + 270$

$= 261.25$

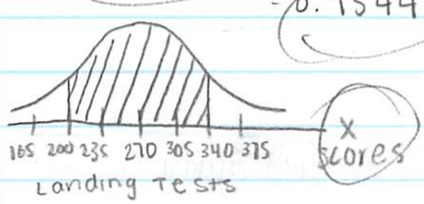
c) $z = \frac{x-\mu}{\sigma}$ $\qquad z = \frac{x-\mu}{\sigma}$

$= \frac{340-270}{35}$ $\qquad = \frac{200-270}{35}$

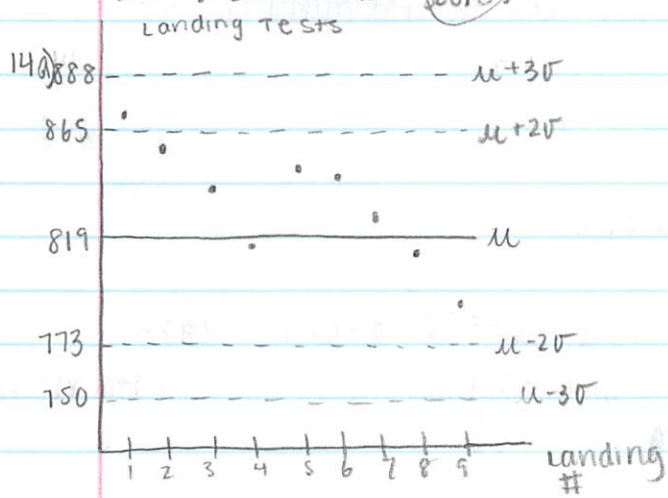
$= 2$ $\qquad = -2$

$P(200 < x < 340) = 0.9772 - 0.0228$

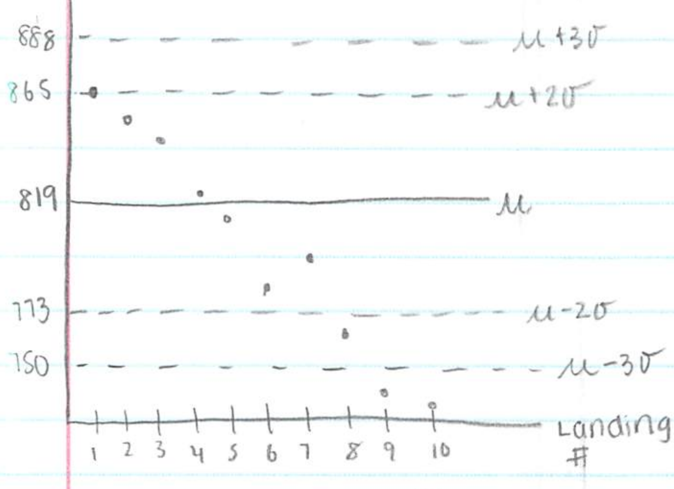
$= 0.9544$



14a)



In control, because there are no out of control signals b)



out of control, because there are 3/3 consecutive points beyond 2σ level (signal III) and 2 points beyond 3σ level (signal I)

18 a) $\mu = (150)(0.68)$ $\quad \sigma = \sqrt{\mu q}$

$= 102$ $\quad = \sqrt{102 \cdot 0.32}$

$= 5.71$

$z = \frac{99.5 - 102}{5.71} = -0.4378$

$P(X > 99.5) = P(Z > -0.44) = 1 - 0.33 = 0.67$

∃x a 67% chance that 100 or more have unlisted numbers.

b) $z = \frac{100.5 - 102}{5.71} = -0.2627$

$P(X < 100.5) = P(Z < -0.26) = 0.3974$

∃x a 39.74% chance that fewer than 100 have unlisted numbers.

c) $\mu = 150 \cdot 0.32$ $\quad \sigma = \sqrt{\mu q}$

$= 48$ $\quad = \sqrt{48 \cdot 0.68}$

$= 5.53$

$z = \frac{65.5 - 48}{5.53} = 3.16$ $\quad z = \frac{49.5 - 48}{5.53} = 0.2712$

$P(49.5 < X < 65.5) = P(0.27 < Z < 3.16) = 0.9992 - 0.6064 = 0.3928$

∃x a 39.28% chance that between 50 and 65 households have listed numbers.

19 a) $\mu = 250 \cdot 0.03 = 7.5$ $\quad \sigma = \sqrt{7.5 \times 0.97} = 2.70$

$z = \frac{4.5 - 7.5}{2.70} = -1.11$

$P(X > 4.5) = P(Z > -1.11) = 1 - 0.1335 = 0.8665$ ∃x a 86.65% chance that 5 or more will have this blood type.

b) $z = -1.11$ $\quad z = \frac{10.5 - 7.5}{2.70} = 1.11$

$P(4.5 < X < 10.5) = P(-1.11 < Z < 1.11) = 0.8665 - 0.1335 = 0.733$ ∃x a 73.3% chance that between 5 and 10 will have this blood type.

SUN

10

Good

7.57 a) Geometric, because we are looking for number of tosses UNTIL the dog gets the ball, it's independent, $p = 0.1$

b) $P(X=2) = (1-p)^{n-1} p$

$= (0.9)^1 (0.1)$

$= 0.09$ ∃x a 9% chance that it will take 2 tosses until sophie catches a ball.

c) $P(X>3) = 1 - P(X=1) - P(X=2) - P(X=3)$

$P(X=3) = (1-0.1)^2 0.1 = 0.081$

$P(X>3) = 1 - 0.1 - 0.09 - 0.081 = \boxed{0.729}$ ∃x a 72.9% chance that it will take more than 3 tosses to catch the ball.

7.58 a) $P(X=1) + P(X=2)$

$0.05 + (0.95)(0.05) = \boxed{0.0975}$ ∃x a 9.75% chance that at most 2 boxes must be purchased.

b) $P(X=4) = (0.95)^3(0.05) = \boxed{0.0429}$ ∃x a 4.29% chance that exactly four boxes must be purchased.

c) $P(X>4) = (1-p)^n$

$= 0.95^4$

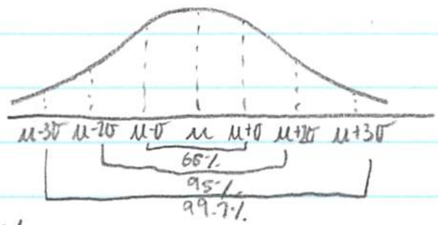
$= \boxed{0.8145}$ ∃x a 81.45% chance that more than 4 boxes must be purchased.

SUN

10

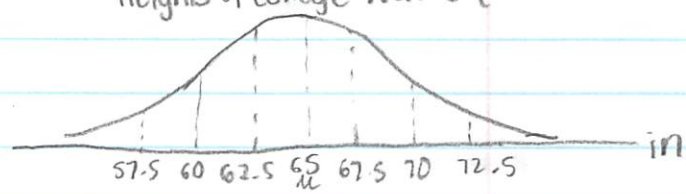
7.1 # 1-7 odd, 8, 10, 11

1. None, not shaped and not unimodal
3. 7-16 has larger σ, $\mu = 10$
   7-17, $\mu = 4$
5. a) 50%
   b) 68%
   c) 99.7%



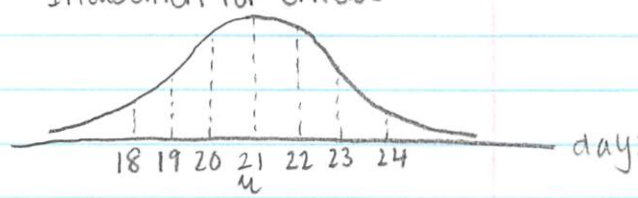
7. a) $P(>65) = 50\%$
   b) $P(<65) = 50\%$
   c) $P(62.5 < W < 67.5) = 68\%$
   d) $P(60 < W < 70) = 95\%$

Heights of college women



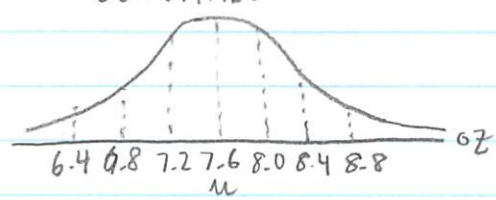
8. a) $P(19 < c < 23) = 95\%$
   b) $P(20 < c < 22) = 68\%$
   c) $P(<21) = 50\%$
   d) $P(18 < c < 24) = 99.7\%$

Incubation for chicks



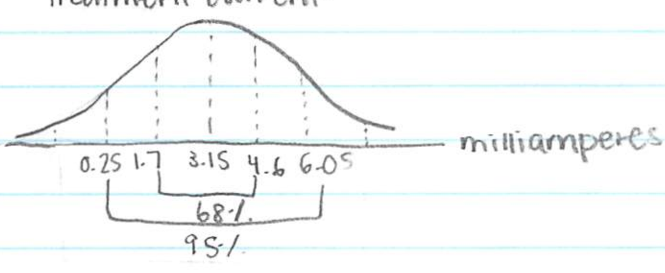
10. a) $P(>8) = 16\%$
    b) $P(<8) = 84\%$
    c) $P(>8) \times 850$
    $0.16 \times 850 = 136$ cups

Soft Drinks



11. a) 1.7 to 4.6 milliamperes
    b) 0.25 to 6.05 milliamperes

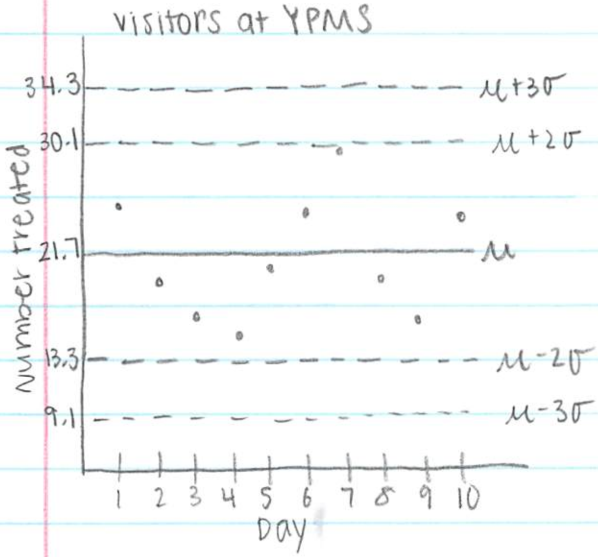
Treatment current



7.1 #12,14

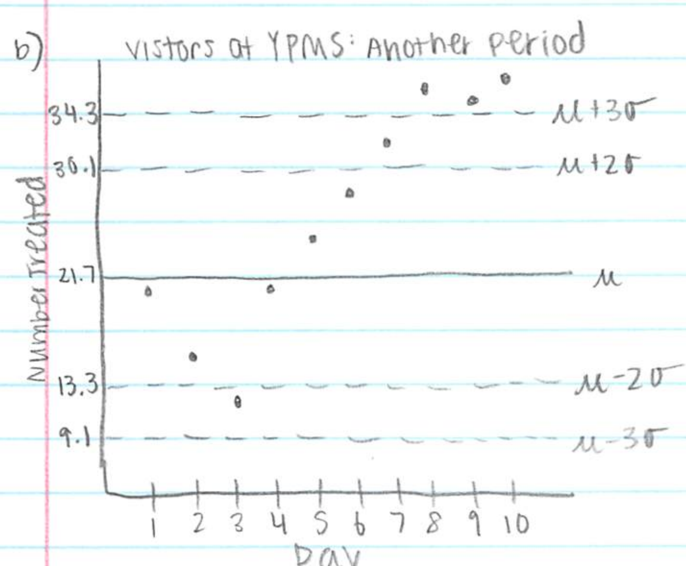
12 a) $\mu = 21.7$

$\sigma = 4.2$



It's in control b/c no pts lie beyond 2σ or 3σ level, and there are not 9 consecutive pts on one side of the center line

b)



out of control. 3 pts fall beyond 3σ level (signal I), and at least 2 out of 3 consecutive pts lie beyond 2σ level on same side of center line (signal II). YPMS might need extra help because they are treating many people and would need more staff to keep patients safe and happy.

14 a) unusually high because many points (11) are above center line. About 268 rooms were expected. 11 consecutive points are above center line (signal II) It could be out of control because of a popular event nearby or vacation season started so more rooms are rented out.

b) Moderately low because more points are below center line than above. 268 rooms are expected to be rented out. One point is beyond 3σ level (out of control signal I). It could be out of control because of a one-day conference at the hotel or it was being used one night for a party.

SUN

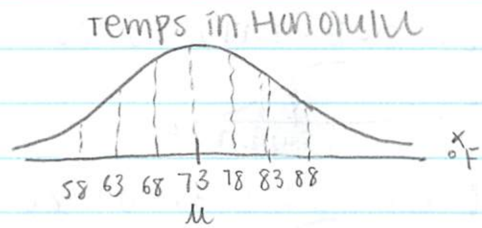
10

7.2 # 1-3, 6, 10-28 E

1. a) Robert, Jan, Linda
   b) Joel
   c) John, Susan
   d) Robert: 172, Jan: 184, Susan: 110, Joel: 150, John: 134, Linda: 182

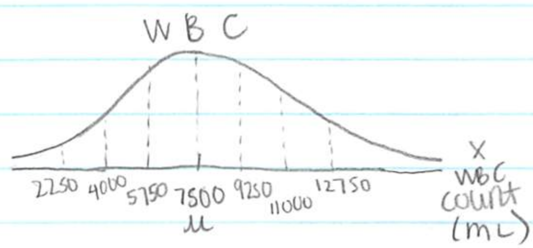
2 a) $45 = z(25) + 51$ $z = -0.24$
   b) $72 = z(25) + 51$ $z = 0.84$
   c) $(75-51)/25 = 0.96$
   d) $(65-51)/25 = 0.24$
   e) $(33-51)/25 = -0.72$
   f) $(55-51)/25 = 0.16$

3. a) $\frac{73-53}{s} = 4$ $-4 < z < 4$



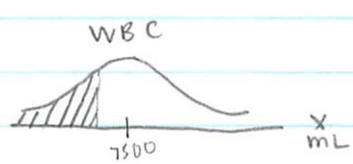
   b) $z < -1.6$
   c) $1 < z$
   d) $81.75°F < x$
   e) $x < 63.5°F$
   f) $64°F < x < 81.25°F$

6. $\mu = 7500$
   $\sigma = 1750$
   a) $\frac{9000-7500}{1750} = 0.8571$



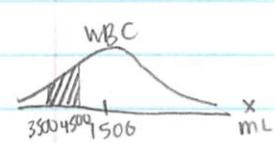
   $0.8571 < z$

b) $\frac{6000-7500}{1750} = -0.8571$



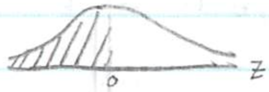
   $z < -0.8571$

c) $-2.29 < z < -1.71$ $\frac{3500-7500}{1750} = -2.29$ $\frac{4500-7500}{1750} = -1.71$

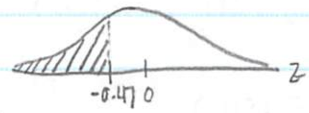


d) $x < 9512.5$ $x = 7500 + (1750)(1.15) = 9512.5$
e) $11332.5 < x$ $x = 7500 + (1750)(2.19) = 11332.5$
f) $7062.5 < x < 9687.5$
g) unusually low, $\frac{2500-7500}{1750} = -2.86$ z scores away from the mean. Only 2.5% of people have WBC count of 2 z-scores lower from mean.

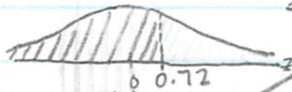
10. $P(z < 0) = 50\%$



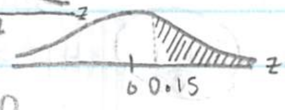
12. $P(z < -0.47) = 0.3192$



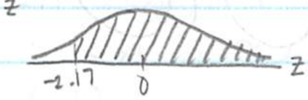
14. $P(z < 0.72) = 0.7642$



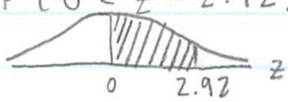
16. $P(z > 0.15) = 1 - 0.5596 = 0.4404$



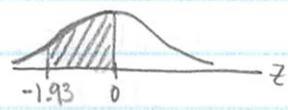
18. $P(z > -2.17) = 1 - 0.0150 = 0.9850$



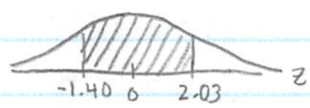
20. $P(0 < z < 2.92) = 0.9982 - 0.5 = \underline{0.4982}$



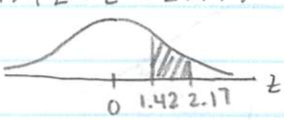
22. $P(-1.93 < z < 0) = 0.50 - 0.0268 = 0.4732$



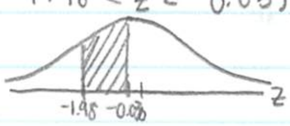
24. $P(-1.40 < z < 2.03) = 0.9788 - 0.0808 = 0.898$



26. $P(1.42 < z < 2.17) = 0.9850 - 0.9222 = 0.0628$



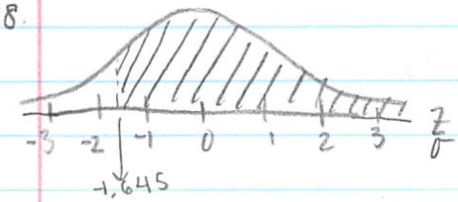
28. $P(-1.98 < z < -0.03) = 0.4880 - 0.0239 = 0.4641$



SUN

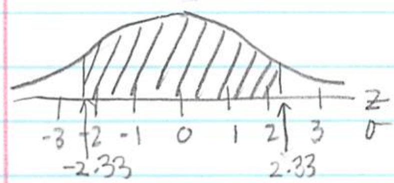
7.3 # 18-20, 23 cd, 24 a, 28

18. $z = -1.645$



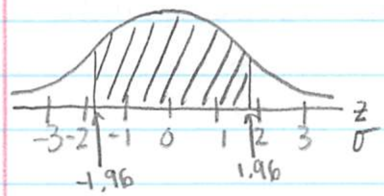
19. $z = 2.33$

$-z = -2.33$



20. $z = 1.96$

$-z = -1.96$

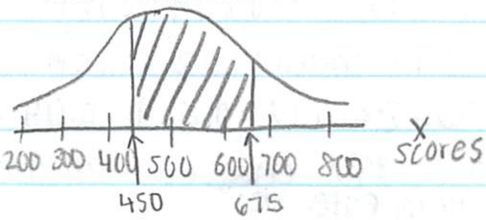


23. c) $\mu = 500, \sigma = 100$

$z = \dfrac{675-500}{100} = 1.75$

$z = \dfrac{450-500}{100} = -0.50$

SAT Math score



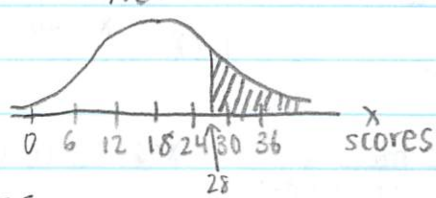
$P(-0.50 < z < 1.75) = 0.9599 - 0.3085 = 0.6514$

∃x a 65.14% chance that a high school's senior's score is between 450 and 675.

d) $\mu = 18, \sigma = 6$

$z = \dfrac{28-18}{6} = 1.67$

ACT



$P(z > 1.67) = 1 - 0.9525 = 0.0475$

∃x a 4.75% chance that a senior's ACT score is higher than 28.

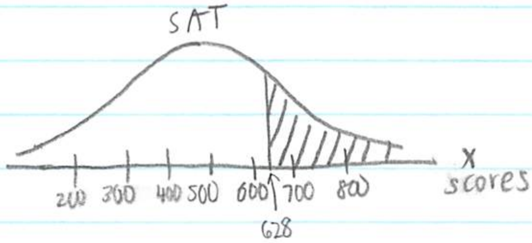
24. a) top 10%

$z > 1.28$

$1.28 = \dfrac{x-500}{100}$

$x = 628$

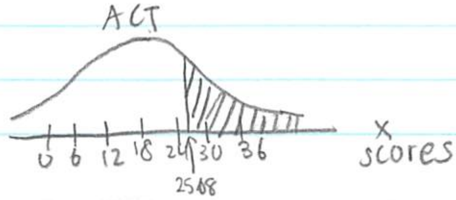
SAT



$1.28 = \dfrac{x-18}{6}$

$x = 25.68 \approx 26$

ACT

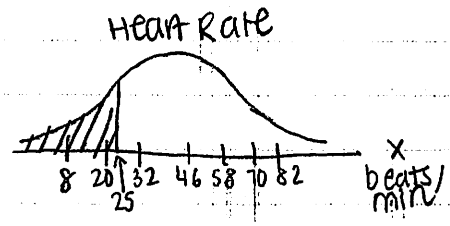


28a) Range = 70 - 22 = 48

4σ will cover 95% range of data values

$\sigma = \frac{48}{4} = 12$

b) $z = \frac{25-46}{12}$

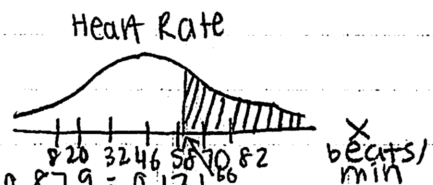


$= -1.75$

$P(z < -1.75) = 0.0401$

∃x a 4.01% chance that heart rate is less than 25 beats/min.

c) $z = \frac{60-46}{12}$



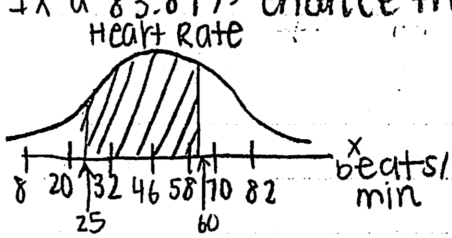
$= 1.17$

$P(z > 1.17) = 1 - 0.879 = 0.121$

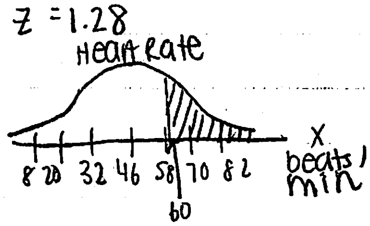
∃x a 12.1% chance that heart rate is greater than 60 beats/min.

d) $P(-1.75 < z < 1.17) = 0.879 - 0.0401 = 0.8389$

∃x a 83.89% chance that heart rate is between 25 and 60 beats/min.



e) $z = 1.28$ $\qquad 1.28 = \frac{x-46}{12}$



$x = 61.36$ beats/min

CATHY SUN

10
Good

6.2 (1-4, 6, 10-12)

1 a) A trial is a flip of a quarter
S = 3 heads, F = tails, n = 3, p = 0.5, q = 0.5
$P(r=3) = C_{3,3}(0.5)^3(0.5)^0 = 0.125$ ∃X a 12.5% chance that 3 out of 3 are heads
b) $P(r=2) = C_{3,2}(0.5)^2(0.5)^1 = 0.375$
∃X a 37.5% chance that 2 out of 3 flips are tails
c) $P(r=2 \text{ or } r=3) = 0.375 + 0.125 = 0.500$
∃X a 50% chance of getting two or more heads
d) $P(r=0) = C_{3,0}(0.5)^0(0.5)^3 = 0.125$
∃X a 12.5% chance of getting exact three tails.

2 a) $P(r=10) = C_{10,10}(0.20)^{10}(0.80)^0$
$= 1.024 \times 10^{-7} = 0.0000001024$
∃X essentially 0% chance that Richard answers all correctly
b) $P(r=0) = C_{10,0}(0.20)^0(0.80)^{10}$
$= 0.1074$
∃X a 10.74% chance that Richard answers all questions incorrectly
c) $P(r \geq 1) = 0.268 + 0.302 + 0.201 + 0.088 + 0.026 + 0.06 + 0.01 = 0.955$
∃X a 95.5% that Richard gets at least 1 correct.
$P(r \geq 1) = 1 - P(r=0)$
$= 1 - 0.1074 = 0.8926$ ∃X a 89.26% that Richard gets at least 1 correct.
d) $P(r \geq 5) = 0.026 + 0.06 + 0.01 = 0.033$
∃X a 3.3% chance that Richard gets at least 5 correct

These answers should be equal but are not, probably due to rounding differences in the table versus calculator

3a) A trial is men answering whether they would marry same woman.
S: yes, F: no, n = 10, p = 0.80, q = 0.20
$P(r \geq 7) = 0.201 + 0.302 + 0.268 + 0.107 = 0.878$
∃X a 87.8% chance that at least 7 will claim to marry same woman
$P(r < 5) = 0.001 + 0.006 = 0.007$
∃X a 0.7% chance that less than 5 men will claim to marry same woman

b) A trial is women answering whether they would marry the same man
S: yes, F: no, n = 10, p = 0.50, q = 0.50

$P(r \geq 7) = 0.075 + 0.023 + 0.04 = 0.172$

∃x a 17.2% chance that at least 7 women would marry same man

$P(r < 5) = 0.001 + 0.01 + 0.044 + 0.117 + 0.205 = 0.377$

∃x a 37.7% chance that less than 5 women would marry same man

4. one trial is whether or not person has had one-time fling.

S = Yes F = No $n = 7$ $p = 0.10$ $q = 0.90$

a) $P(r = 0) = C_{7,0}(0.10)^0(0.90)^7 = 0.4783$

∃x a 47.83% chance that no one has done a one-time fling.

b) $P(r \geq 1) = 0.372 + 0.124 + 0.023 + 0.03 = 0.549$

∃x a 54.9% chance that at least 1 person has done a one-time fling

c) $P(r \leq 2) = 0.478 + 0.372 + 0.124 = 0.974$

∃x a 97.4% chance that no more than 2 people have done a 1-time fling.

6. Trial ~~is whether tie is too~~ tight.

S = Yes F = NO $n = 20$ $p = 0.10$ $q = 0.90$

a) $P(r \geq 1) = 1 - P(0) = 1 - 0.122 = 0.878$

∃x a 87.8% chance that at least one tie is too tight.

b) $P(r > 2) = 1 - P(r \leq 2) = 1 - 0.122 - 0.270 - .285 = 0.323$

∃x a 32.3% chance that more than 2 ties are too tight.

c) $P(r = 0) = C_{20,0}(0.10)^0(0.90)^{20} = 0.1216$

∃x a 12.16% chance that none is too tight.

d) ~~S = No F = Yes n = 20 p = 0.90 q = 0.10~~

$P(r \leq 2) = 0.285 + 0.270 + 0.122 = 0.677$

∃x a 67.7% chance that at least 18 are not too tight.

10. one trial is one taste test. S = picks tasty bean, F = doesn't,

$n = 4$, $p = 0.20$, $q = 0.80$

a) $P(r = 4) = C_{4,4}(0.20)^4(0.80)^0 = 0.0016$

∃x a .16% chance that all 4 tasters choose tasty bean.

b) $P(r = 0) = C_{4,0}(0.20)^0(0.80)^4 = 0.4096$

∃x a 40.96% chance that none choose tasty bean

c) $P(r \geq 3) = C_{4,3}(0.20)^3(0.80)^1 + (0.20)^4 = 0.0272$

∃x a 2.72% chance that at least 3 choose tasty bean

Cathy Sun

9

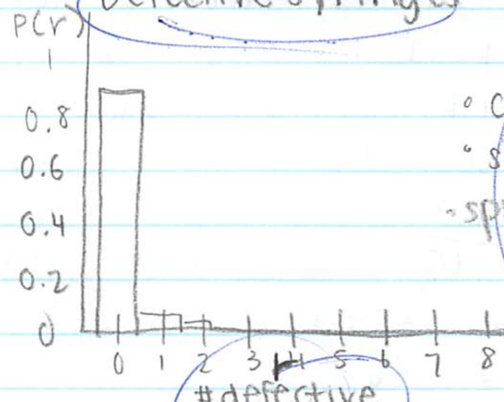
2 a) II
b) I
c) III
d) IV
e) The graph is more symmetrical when $p = 0.50$. When p is close to 1, it's skewed right, skewed left when p is close to 0

6.3 #24, 5, 7, 8

4 a) S = defective, F = not defective, $n = 8$, $r = 0-8$, $p = 0.01$, $q = 0.99$

$n=8$	$r=0$	0.9227
	1	0.07457
	2	0.002636
	3	5.33×10^{-6}
	4	6.72×10^{-7}
	5	5.43×10^{-9}
	6	2.74×10^{-11}
	7	7.92×10^{-14}
	8	$1 \times 10^{-16} \approx 0$

Defective Syringes



Ana
- center: $\mu = 0.08$
- shape: skewed right
- spread: $\frac{0.28}{0.08} \times 100 =$ 350%, WILD
- outliers: N/A

b) $\mu = np$
$= 8 \times 0.01 = 0.08$ defective syringes

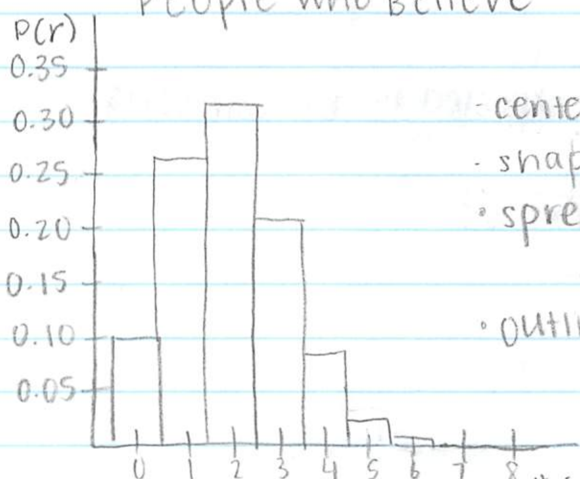
c) $P(r=0) = C_{8,0}(0.01)^0(0.99)^8 = 0.9227$ $+ P(r=1) = C_{8,1}(0.01)^1(0.99)^7 = 0.07457$ } ∴ ~~92.27%~~ .998 chance that the batch will be accepted

d) $\sigma = \sqrt{\mu q}$
$= \sqrt{0.08 \times 0.99}$
$= 0.2814$

5 a) S = believe, F = don't believe, $n = 8$, $r = 0-8$, $p = 0.25$, $q = 0.75$

$n=8$	$r=0$	0.100
	1	0.267
	2	0.311
	3	0.208
	4	0.087
	5	0.023
	6	0.004
	7	0
	8	0

People who Believe



Ana
- center: $\mu = 2$
- shape: skewed right
- spread: $\sigma = \frac{1.22}{2} \times 100 = 61\%$, moderate
- outliers: N/A

b) $\mu = np$
$= 8 \times 0.25 = 2$ non-believers

c) $\sigma = \sqrt{npq}$
$= \sqrt{\mu q}$
$= \sqrt{1.5} = 1.22$

7 a) Japan
$P(r=7) = C_{7,7}\,(0.95)^7(0.05)^0 = 0.6983$ ∃x a 69.83% chance that all 7 defendents are found guilty in Japan
U.S.
$P(r=7) = C_{7,7}\,(0.60)^7(0.40)^0 = 0.02799$
∃x a 2.799% chance that all 7 defendants are found guilty in U.S.

b) Japan

$\mu = np$
$= 7 \times 0.95$
$= 6.65$

$\sigma = \sqrt{\mu q}$
$= \sqrt{6.65 \times 0.05}$
$= 0.5766$

U.S.

$\mu = np$
$= 7 \times 0.60$
$= 4.20$

$\sigma = \sqrt{\mu q}$
$= \sqrt{4.20 \times 0.40}$
$= 1.30$

8. s = extroverted, F = not extroverted, $n = 6$, $p = 0.45$, $q = 0.55$

1. $P(r=6) = C_{6,6}\,(0.45)^6(0.55)^0 = 0.008304$
∃x a 0.8304% chance that all 6 are extroverts.

2. $P(r=0) = C_{6,0}\,(0.45)^0(0.55)^6 = 0.02768$
∃x a 2.768% chance that none is an extrovert.

3. $1 - P(r \leq 1) = 1 - 0.028 - 0.136 = 0.836$
∃x a 83.6% chance that at least two of the professors are extroverts

4. $\mu = np$
$= 6 \times 0.45$
$= 2.7$ expected to be extroverts

$\sigma = \sqrt{\mu q}$
$= \sqrt{2.7 \times 0.55}$
$= 1.22$

SUN

6 RVW #5-7

5a) S = sewing time, F = not sewing time, $n = 16$, $p = 0.50$, $q = 0.50$

$P(r \geq 12) = 0.0384$ ∃x a 3.84% that 12 or more are sewing time for drug dealing

b) $P(r \leq 7) = 0.4018$ ∃x a 40.18% that fewer than 7 inmates are sewing time for drug dealing.

c) $\mu = np$ $\quad\quad$ $\sigma = \sqrt{\mu q}$

$= 16 \times 0.5 = 8$ $\quad\quad$ $= \sqrt{8 \times 0.5} = 2$

$CV = \frac{2}{8} \times 100 = 25\%$ (10W)

6. $n = 200$ $\quad p = 0.8$ $\quad q = 0.2$

$\mu = np$

$= 200 \times 0.8$

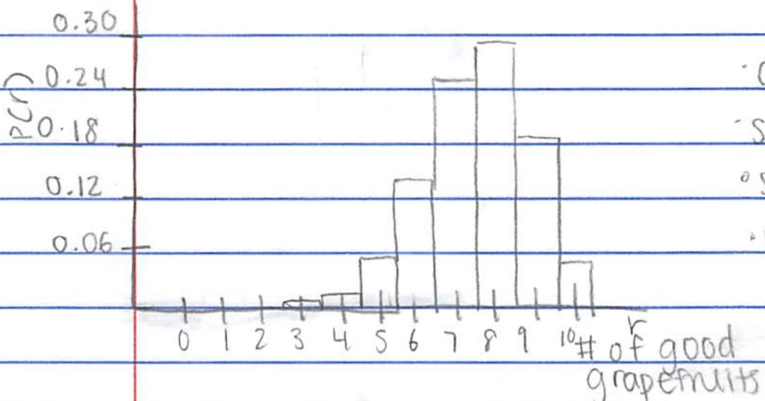
$= 160$ expected to arrive on time

$\sigma = \sqrt{\mu q}$

$= \sqrt{160 \times 0.2}$

$= 5.66$

7a) Grapefruits



$n = 10$ $\quad p = 0.75$ $\quad q = 0.25$

Ana

- center = $\mu = 150$
- shape = skewed left
- spread = $CV = \frac{6.12}{150} \times 100 = 4.08\%$ 10W
- outliers: X

b) $P(r \geq 9) = 0.2440$

$P(r \geq 1) \approx 1$

c) $\mu = np$

$= 200 \times 0.75$

$= 150$ expected # of good grapefruits

d) $\sigma = \sqrt{\mu q}$

$= \sqrt{150 \times 0.25}$

$= 6.12$

Use the following information for questions 5–6.

A company claims that the number of defective items manufactured during each run of making 100 of their products is independent of the number from other runs and that the proportion of defectives is no more than 4%. Assume that the proportion of defectives for each run is .04.

5. What is the probability that there will be no defectives on the next run?
   (A) .034
   (B) .051
   (C) .017
   (D) .0085
   (E) None of these.

6. The distribution of the number of defectives in each run of the manufacturing process has an expected value of 4 and a standard error of 1.96. Which of the following is a correct interpretation of these values?
   (A) There is a probability of .95 that the number of defectives will be within 3.92 of 4.
   (B) There is a .0196 probability that the number of defectives will equal 4.
   (C) The distribution is symmetric and mound-shaped with the center at 4.
   (D) The rule of thumb dealing with percentages of data with 1, 2, and 3 standard deviations of the mean cannot be applied since this distribution could be severely skewed.
   (E) None of these is a correct interpretation.

FREE-RESPONSE QUESTIONS

Open-Ended Questions

1. Create and display probability plots of each binomial probability distribution specified. The horizontal axis is the number of successes: 0–10.
   a. $n = 10; p = .3$
   b. $n = 10; p = .5$
   c. $n = 10; p = .8$

Compare symmetry versus skewness of your probability plots and discuss how they relate to the value of p.

2. Find the mean and standard deviation of the distributions in questions 1a, 1b, and 1c.

The concept of expected value can be applied to games and the determination of strategies.

3. In the game of roulette, there are 18 black numbers, 18 red numbers, and 2 green numbers. If you bet $1 on a color and win, you receive $1. Consider your "gain" to be +$1 if you win and −$1 if you lose.
   a. How much would you expect to gain on *each* play?
   b. If you were to bet on a color 100 times, how much would you expect to gain?

4. There are two soda machines in your school. A can of soda costs 75 cents from Machine A and Machine A works 100% of the time. A can of soda costs 50 cents from Machine B and Machine B works 60% of the time. When Machine B does not work, it eats your money. You determine to try the following strategy:

   Try machine B. If it does not work, go to Machine A and purchase your soda from it.

   Evaluate this strategy in the long run.

5. If the Cubs are expected to win 70% of their games,
   a. How many games would you expect them to win during a 162-game schedule?
   b. How much error would you expect in this estimate? (Hint: Consider standard error.)

6. In roulette, there are 38 numbers, 00, 0, and 1–36. If you bet $1 on a number and win, you receive an additional $35. How much do you expect to win or lose on each play if you bet on a number?

Section 5.5 Probability Distributions of Discrete Random Variables

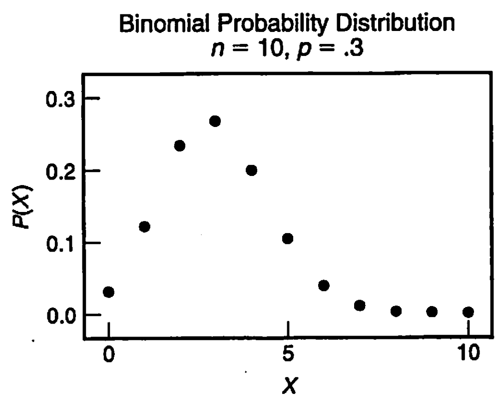
MULTIPLE-CHOICE QUESTIONS (page 122)

1. **B** I: sum of the probabilities is greater than 1; IV: one of the probabilities is negative.
2. **B** Only II and III are probability distributions, and the values of III are negative.
3. **C** Only II and III are probability distributions and the probabilities are concentrated in a smaller range of values in III.
4. **B** For example, the graph of a geometric distribution is not symmetric.
5. **C** $\binom{100}{0}(.04^0)(.96^{100}) = binompdf(100, .04, 0) = .0169$
6. **D**

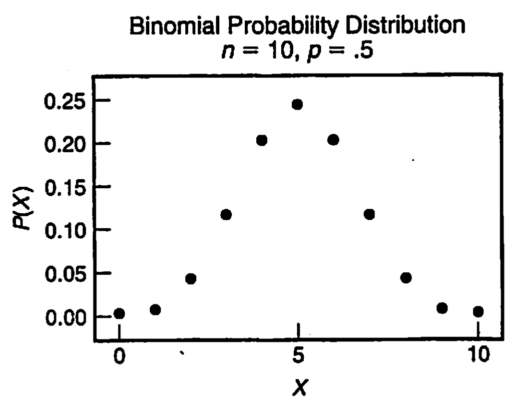
FREE-RESPONSE QUESTIONS

Open-Ended Questions (page 123)

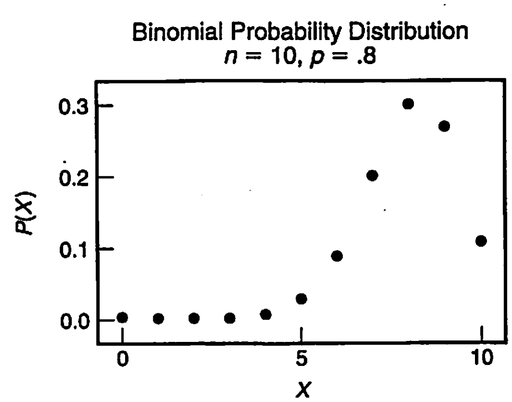
1. a.



b.



c.



2. a. mean = 3; standard deviation = 1.45
   b. mean = 5; standard deviation = 1.58
   c. mean = 8; standard deviation = 1.26
3. a. Expected value $= (+1)\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right)$ .05
   b. Expect to *lose* 5.26 cents for each dollar bet. Playing 100 times, one would expect to lose 100($.0526) = $5.26.
4. Expected value using the strategy = (.50)(.6) + (1.25)(.4) = .80; On average, using the strategy wil cost 80 cents. Therefore it is better to use Machine each time.
5. a. Expected value = 162.7(.7) = 113.4
   b. Standard error $= \sqrt{162.7(.7)(.3)} = 5.83$ assumi the probability (.7) is the same for each game a that each game is independent.
6. Expected value $= (+35)\left(\frac{1}{38}\right) + (-1)\left(\frac{37}{38}\right) = -.052$

Mean and Standard Deviation of a General Discrete Random Variable

For a general discrete probability distribution, we have the following formulas for the mean and variance of the distribution:

If the sample space of an event contains the values $x_1, x_2, \ldots, x_n$, having probabilities of occurring of $p_1, p_2, \ldots, p_n$, then the mean of the probability distribution is

$$\mu_x = x_1p_1 + x_2p_2 + \ldots + x_np_n$$
$$= \sum_{i=1}^{n} x_ip_i$$

and the variance is

$$\sigma^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \ldots + (x_n - \mu_x)^2 p_n$$
$$= \sum_{i=1}^{n} (x_i - \mu_x)^2 p_i$$

The mean of a probability distribution is denoted by the Greek letter μ (mu), rather than $\bar{x}$ as the mean for sample data. Similarly, the variance of the distribution is denoted by the Greek letter σ^2 (sigma) instead of s^2 for sample data. Consequently, σ is the symbol for the standard deviation of the probability distribution.

Expected Value and Standard Error

In this context, the mean is usually given the name **expected value**; in some books, the standard deviation is called the **standard error**.

EXAMPLE 2

Determine the expected number of heads and the standard error in three coin flips using the definition. Verify by using the binomial formulas.

Solution

Outcome: Number of heads (x_i)	Probability (p_i)	Product, x_ip_i	$(x_i-\mu)^2p_i$
0	${}_3C_0(.5)^0(.5)^3 = .125$	0	.28125
1	${}_3C_1(.5)^1(.5)^2 = .375$	.375	.09375
2	${}_3C_2(.5)^2(.5)^1 = .375$	.750	.09375
3	${}_3C_3(.5)^3(.5)^0 = .125$	.375	.28125

$$\text{Expected Value} = \mu = \sum_{i=0}^{3} x_ip_i = 0 + .375 + .750 + .375 = 1.5 \text{ heads}$$

$$\text{Standard Error} = \sigma = \sqrt{\sum_{i=0}^{3}(x_i - \mu)^2 p_i}$$
$$= \sqrt{.28125 + .09375 + .09375 + .28125} \approx .866$$

This distribution is binomial with $n = 3$ and $p = .5$. Therefore, by the binomial formulas,

$$\text{Expected Value} = np = 3(.5) = 1.5$$
$$\text{Standard Error} = \sqrt{np(1-p)} = \sqrt{(1.5)(.5)} = .866$$

Expected value is also a useful tool for decision-making in business.

EXAMPLE 3

The Prova Insurance Company offers a homeowner's fire insurance policy. Their records indicate that during a year the company will pay out the following amounts for claims due to fires:

$100,000 with a probability of .0004
$50,000 with a probability of .002
$25,000 with a probability of .008
$10,000 with a probability of .015
$5,000 with a probability of .03
$1,000 with a probability of .07

What should the company charge for each fire policy so that the sum of the premiums will cover the expenses due to claims?

Solution

The expected value of the company's payments is

$$100{,}000(.0004) + 50{,}000(.002) + 25{,}000(.008) + 10{,}000(.015) + 5{,}000(.03) + 1{,}000(.07) = 710$$

This $710 payment is an average over the long run. Remember, the company will not pay exactly this amount on each policy. The standard deviation gives us an idea of the spread of the payments.

To cover its expected payments, the company should charge a minimum of $710 for each policy.

Review Exercises

MULTIPLE-CHOICE QUESTIONS

Use the following distributions to answer questions 1–3.

I: X	I: p	II: X	II: p	III: X	III: p	IV: X	IV: p
0	.5	10	.2	−2	0	1	−.1
1	.2	12	.2	−5	0	2	.2
2	.2	15	.3	−10	.3	3	.3
3	.1	19	.2	−14	.3	4	.3
4	.1	25	.1	−20	.4	5	.3

1. Which of the tables does not constitute a legitimate discrete probability distribution?
   (A) I only
   (B) I and IV
   (C) I, II, and IV
   (D) III only
   (E) I and III

2. Which of the probability distributions has the greatest mean?
   (A) I
   (B) II
   (C) III
   (D) IV
   (E) There is no maximum.

3. Which of the probability distributions has the smallest standard deviation?
   (A) I
   (B) II
   (C) III
   (D) IV
   (E) There is no minimum.

4. Which of the following is not true of discrete probability distributions?
   (A) The sum of the probabilities is 1.
   (B) The graph of the distribution exhibits symmetry.
   (C) The value of the standard deviation can be less than, equal to, or greater than the value of the mean.
   (D) Each probability in the distribution must be greater than or equal to 0.
   (E) All of these are true statements.

5.5 Probability Distributions of Discrete Random Variables

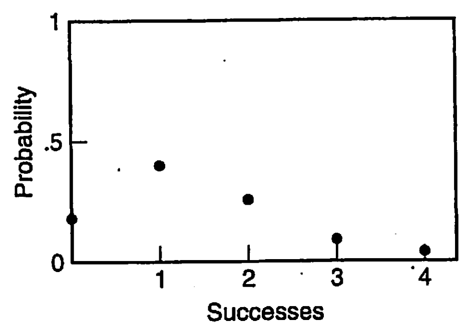
A **probability distribution** for an event consists of a list of all outcomes in the event and their probabilities. We will first consider the case of the probability distribution of a discrete random variable with a finite number of outcomes, using the binomial case. We will then treat a probability distribution of a continuous random variable.

The Probability Distribution of a Binomial Variable

The probability distribution of a binomial variable with $n = 4$ and $p = .3$, $X \sim \mathit{Binomial}(4, .3)$ is displayed below:

Successes	Expression	Value
0	${}_4C_0(.3)^0(.7)^4$	.2401
1	${}_4C_1(.3)^1(.7)^3$	.4116
2	${}_4C_2(.3)^2(.7)^2$	.264
3	${}_4C_3(.3)^3(.7)^1$	.0756
4	${}_4C_4(.3)^4(.7)^0$	.0081
	Sum	1

Probability Plot



Technology Note: Using a graphing calculator with the distribution (*DISTR*) command provides all of the binomial probabilities with one command. For example, using a TI-83 and the command *binompdf(4, .3)* returns {.2401, .4116, .2646, .0756, .0081} representing, in order, all of the binomial probabilities for successes $r = 0, 1, 2, 3,$ and 4. ■

Technology Note: In order to view the probability plot of a binomial distribution using the TI-83, follow these steps:

Instruction	TI-83 Command
1. Load the possible values of the numbers of successes in L_1.	*seq*(*x*, *x*, 0, 4, 1) $\rightarrow L_1$ [*seq(* is on the *LIST OPS* menu.]
2. Calculate and load the binomial probabilities for each number of successes into L_2.	L_2 = *binompdf*(4, .3, L_1) [L_2 is in the *STAT EDIT* window]
3. Plot the scatterplot with *X*-list L_1 and *Y*-list L_2	Set *STAT PLOT 1* to scatterplot, L_1, L_2 and press *ZOOM STAT*.

■

Mean and Standard Deviation of a Binomial Probability Distribution

You will discover in the exercises that binomial distributions, discrete though they are, are somewhat symmetric. Therefore, it is reasonable to consider the mean and standard deviation of a binomial random variable.

Binomial Formulas

The distribution of the outcomes of a binomial random variable with n trials when the probability of a success on one trial is p has

Mean = np

Standard Deviation = $\sqrt{np(1-p)}$

EXAMPLE 1

Determine the mean and standard deviation of a binomial random variable with $n = 4$ and $p = .3$.

Solution

Mean = $np = (4)(.3) = 1.2$

This means that if we have a probability of success of .3 and we make four attempts, we expect to have 1.2 successes.

Standard Deviation = $\sqrt{np(1-p)} = \sqrt{(1.2)(.7)} \approx .917$

Consider these summary statistics with regard to the distribution considered at the beginning of this section. Even though a binomial distribution is discrete, the mean is still the "balance point" of the distribution.

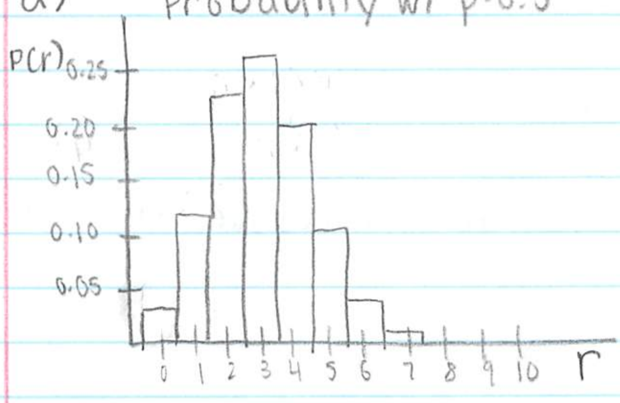
Technology Note: Graphing calculators can be used to verify these formulas. For the TI-83, load all possible numbers of successes $(0, 1, 2, \ldots, n)$ into L_1. Load the corresponding binomial probabilities into L_2. Executing the command for one-variable statistics, *1-Var Stats* L_1, L_2 will display the summary statistics for the binomial probability distribution requested. (Note that the standard deviation for a binomial distribution is listed as σ_x in the output of the *1-Var Stats* command.) ■

SMN
PIXLER

Amsco

1. B
2. B
3. C
4. B
5. C
6. P

1. a) probability w/ p:0.3

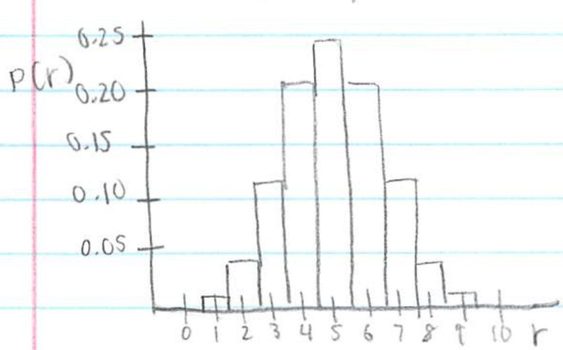


Ana
- center = $\mu = 3$
- shape: skewed right
- spread: $CV = \frac{1.45}{3} \times 100 = 48\%$, mod
- outliers: X

$\mu = np = 3$

$\sigma = \sqrt{\mu q} = \sqrt{3 \times 0.7} = 1.45$

b) probablity w/ p: 0.5

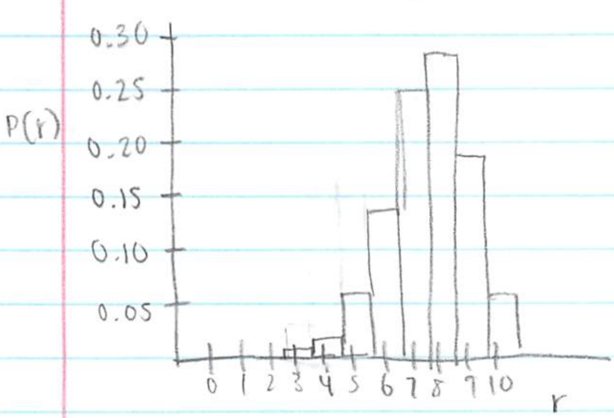


Ana
- center: $5 = \mu$
- shape: symmetric
- spread: $CV = \frac{1.58}{5} \times 100\% = 31.6\%$, mod low
- outliers: X

$\sigma = \sqrt{\mu q}$

$= \sqrt{5 \times 0.5} = 1.58$

c) probabillty w/ p:0.8



Ana
- center: $\mu = 8$
- shape: skewed left
- spread: $CV = \frac{1.26}{8} \times 100\% = 15.75\%$, low
- outliers: X

$\mu = np$

$= 10 \times 0.8 = 8$

$\sigma = \sqrt{\mu q}$

$= \sqrt{8 \times 0.2}$

$= 1.26$

As p gets closer to 0.5, the probability plots get more symmetric. A higher p is skewed left, while a lower p creates skewed right.

2\. 1a) $\mu = np$ $\quad\quad \sigma = \sqrt{\mu q}$

$= 10 \times 0.3 = 3$ $\quad\quad = \sqrt{3 \times 0.7} = 1.45$

b) $\mu = np$ $\quad\quad \sigma = \sqrt{\mu q}$

$= 10 \times 0.5 = 5$ $\quad\quad = \sqrt{5 \times 0.5} = 1.58$

c) $\mu = np$ $\quad\quad \sigma = \sqrt{\mu q}$

$= 10 \times 0.8 = 8$ $\quad\quad = \sqrt{8 \times 0.2} = 1.26$

3a) gain $= (1)\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right)$

$= -0.05263$

b) $0.05263 \times 100 = \$5.26$

4\. $\mu = \overset{np}{(0.50)}\overset{+}{(0.60)} + \overset{np}{(1.25)}(0.4)$

$= 0.80$

It will cost 80 ¢ on average, so might as well use machine A since a soda only costs 75 ¢.

5a) $\mu = np$

$= 162 \times 0.70 = 113.4$ games

b) $\sigma = \sqrt{\mu q}$

$= \sqrt{113.4 \times 0.3} = 5.83$

6\. $\mu = (35)\left(\frac{1}{38}\right) + (-1)\left(\frac{37}{38}\right)$

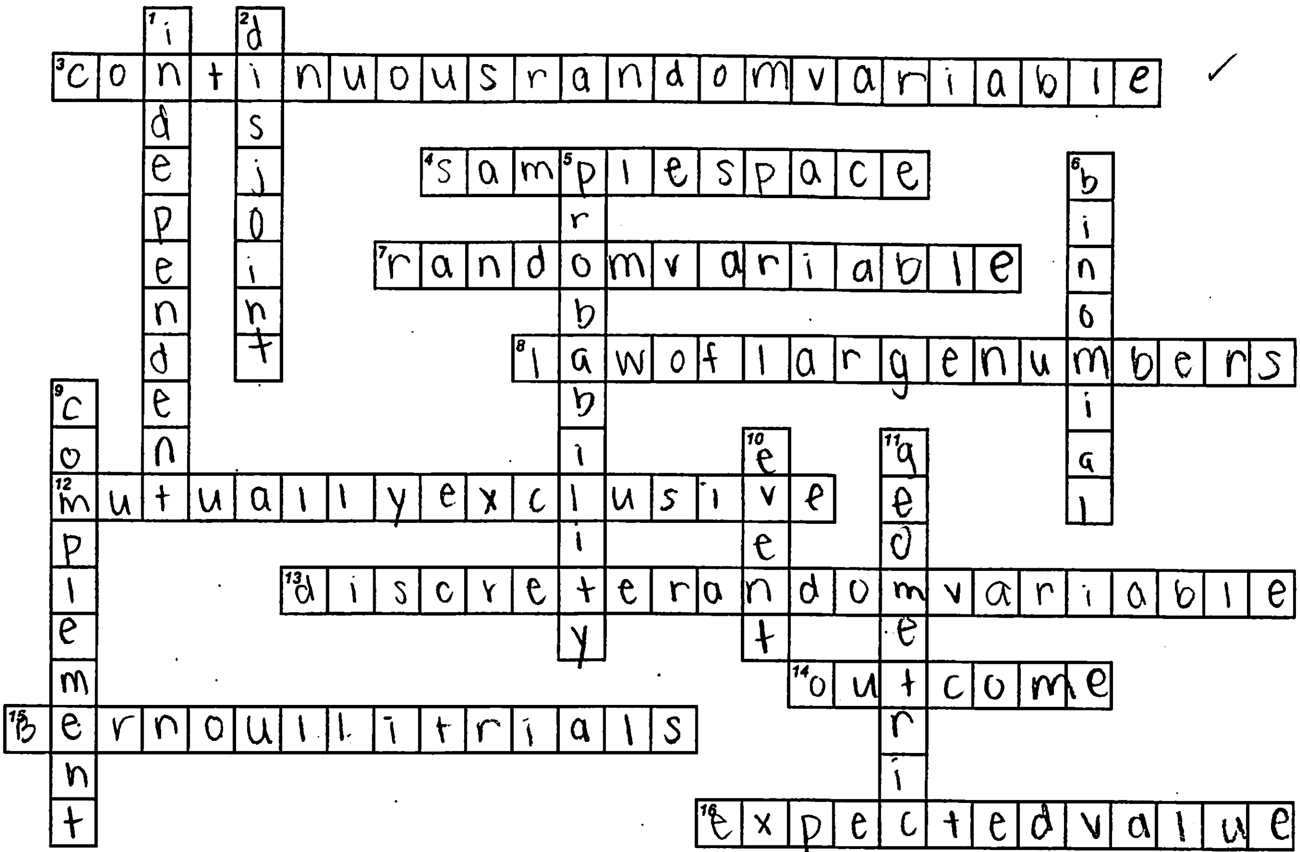
$= -0.05263$

SUN

10

Randomness & Probability

Advanced Placement Statistics



Stats: Modeling the World, Chapters 14-17

ACROSS

3 random variable that can take any numeric value within a range of values
4 collection of all possible outcome values
7 variable that assumes any of several different values as a result of some random event
8 the long-run relative frequency of repeated independent events settles down the true relative frequency as the number of trials increases
12 disjoint
13 random variable that can take one of a finite number of a distinct outcome
14 value measured, observed, or reported for an individual instance for a trial
15 situation where there are two possible outcomes, the probability of success is constant, and the trials are independent
16 the theoretical long-run average value

DOWN

1 relationship between events if knowing one event occurs does not alter the probability that the other event occurs
2 relationship between two events if they share mo outcomes in common
5 a number between 0 and 1 that reports the likelihood of an event's occurrence
6 probability model appropriate for a random variable that counts the number of successes in a fixed number of Bernoulli trials
9 set of outcomes that are not in the specified event
10 collection of outcomes
11 probability model appropriate for a random variable that counts the number of Bernoulli Trials until the first success.

SMN

+2ec

P253 Hawaii

Honolulu 1) $n = 31$, $p = 0.60$, $q = 0.40$

r	$P(r)$
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0.00002072
8	0.00009324
9	0.0003574
10	0.001179
11	0.003378
12	0.008444
13	0.01851
14	0.03570
15	0.06069
16	0.09104
17	0.1205
18	0.1406
19	0.1443
20	0.1298
21	0.1020
22	0.06956
23	0.04083
24	0.02041
25	0.008574
26	0.002968
27	0.0008244
28	0.0001767
29	0.00002741
30	0
31	0

2) Honolulu

$\mu = np = 31 \times 0.6 = 18.6$ $\quad \sigma = \sqrt{\mu q} = \sqrt{18.6 \times 0.4} = 2.73$

Juneau

$\mu = np = 31 \times 0.18 = 5.58$ $\quad \sigma = \sqrt{\mu q} = \sqrt{5.58 \times 0.82} = 2.14$

Seattle

$\mu = np = 31 \times 0.24 = 7.44$ $\quad \sigma = \sqrt{\mu q} = \sqrt{7.44 \times 0.76} = 2.38$

Hilo

$\mu = np = 31 \times 0.36 = 11.16$ $\quad \sigma = \sqrt{\mu q} = \sqrt{11.16 \times 0.64} = 2.67$

Las Vegas

$\mu = np = 31 \times 0.75 = 23.25$ $\quad \sigma = \sqrt{\mu q} = \sqrt{23.25 \times 0.25} = 2.41$

Phoenix

$\mu = np = 31 \times 0.77 = 23.87$ $\quad \sigma = \sqrt{\mu q} = \sqrt{23.87 \times 0.23} = 2.34$

3. $P(r \le 7) = 0.8184$ ∃x a 81.84% that Juneau has at most 7 clear days.

4. $P(5 \le r \le 10) = P(r \le 10) - P(r \le 4) = 0.7945$ ∃x a 79.45% that Seattle has 5-10 clear days.

5. $P(r \ge 12) = 1 - P(r \le 11) = 0.4426$ ∃x a 44.26% chance that Hilo will have at least 12 clear days.

6. $P(r \ge 20) = 1 - P(r \le 19) = 0.9637$ ∃x a 96.37% chance that Phoenix will have 20 or more clear days.

7. $P(20 \le r \le 25) = P(r \le 25) - P(r \le 19) = 0.7592$

∃x a 75.92% chance that Las Vegas has 20-25 clear days.