cathy sun

10

5.2 1-4, 8, 9, 15-20

1a) 0.10 + 0.10 = 0.20, yes because it's just drawing 1 candy, can't get both

b) 0.20 + 0.20 = 0.40, yes because there is no overlap

c) ~~1 - 0.20 = 0.80~~

P(G ∪ B) = P(G) + P(B)

2a) 0.20 + 0.10 = 0.30, yes b/c there is no overlap why?

P(Y ∪ R) = P(Y) + P(R)

b) 0.20 + 0.20 = 0.40, yes b/c there is no overlap

P(P') =

c) 1 - 0.20 = 0.80

Yes because the distributions are different.

P(G ∪ B) = P(G) + P(B)

3a) 0.166 + 0.166 = 0.332, yes, there is no overlap

P(Y ∪ R) = P(Y) + P(R)

b) 0.166 + 0.166 = 0.332, yes, only picking 1 so probabilities ~~aren't~~ don't overlap

P(P') =

c) 1 - 0.166 = 0.834

The results are different because the candies are distributed differently

4a) $111/288 = 0.3854$

b) $30/288 + 33/288 + 18/288 = 0.2813$

c) $111/288 + 96/288 + 30/288 = 0.8229$

d) $96/288 + 30/288 + 33/288 = 0.5521$

e) $18/288 = 0.0625$

8a) P(1 and 6) or P(2 and 5) or P(3 and 4) or P(6 and 1) or P(5 and 2) or P(4 and 3)

$(\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) = 0.1667$

b) P(5 and 6) or P(6 and 5)

$(\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) = 0.05556$

c) 0.1667 + 0.05556 = 0.2223, yes b/c you cannot get a sum of 7 and 11.

9a) NO, the probability when drawing the second card is changed because there are only 51 cards left

b) P(Ace) · P(King | Ace)

$\frac{4}{52} \cdot \frac{4}{51} = 0.006033$

c) 0.006033

d) 0.006033 + 0.006033 = 0.012066

15a) $P(T \text{ and } WA) = P(T) \cdot P(WA) =$
$0.9 \times 0.07 = 0.063$

b) $P(L \text{ and } C) = P(L) \cdot P(C) =$
$0.1 \times 0.72 = 0.072$

c) $P(T \text{ and } WA) = P(T) \cdot P(WA)$
$0.50 \times 0.07 = 0.035$

$0.50 \times 0.72 = 0.36$

d) $0.15 \times 0.07 = 0.0105$

$0.85 \times 0.72 = 0.612$

16a) $P(B) = 0.3$

$P(B \text{ given } A) = 0.72$

$P(B, \text{ given not } A) = 0.07$

$0.3 = P(A)\,0.72 + [(1 - P(A)](0.07)$

$0.3 = 0.72\,P(A) + 0.07 - 0.07\,P(A)$

$P(A) = 0.3538 = 35.38\%$ lies

b) $0.7 = P(A)\,0.72 + (1 - P(A))\,0.07$

$P(A) = 0.9692 = 96.92\%$ lies

17a) $P(S) = \frac{686}{1160} = 0.5914$ $P(S \mid A) = \frac{270}{580} = 0.4655$ $P(S \mid Pa) = \frac{416}{580} = 0.7172$

b) No, there can be a passive approach sale

c) $P(A \text{ and } S) = \frac{270}{1160} = 0.2328$

$P(Pa \text{ and } S) = \frac{416}{1160} = 0.3586$

d) $P(N) = \frac{474}{1160} = 0.4086$

$P(N \mid A) = \frac{310}{580} = 0.5345$

e) No, can have an aggressive approach + no sale

f) $P(A \text{ or } S) = \frac{580}{1160} + \frac{686}{1160} - \frac{270}{1160} = 0.8586$

18a) $\frac{110}{136} = 11/13$

b) $\frac{20}{136} = 2/13$

c) $\frac{50}{70} = 5/7$

d) $\frac{20}{70} = 2/7$

e) $\frac{110}{200} = 11/20$

f) $\frac{20}{200} = 1/10$

19a) $\frac{72}{154} = 0.4675$

b) $\frac{82}{154} = 0.5325$

c) $\frac{79}{116} = 0.6810$

d) $\frac{37}{116} = 0.3190$

e) $\frac{72}{270} = 0.2667$

f) $\frac{82}{270} = 0.3037$

Cathy Sun

AMSCO permutations

1. $C_{5,1} \cdot C_{7,1} \cdot C_{4,1} = 140$

2. $\underbrace{\_\ \_\ \_}_{\text{letters}}\ \underbrace{\_\ \_\ \_}_{\#s}$

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$

3. _ _ _

$41 \cdot 41 \cdot 41 = 68921$

4. $\_\ \underset{\text{even}}{\_}\ \_\ \underset{\text{even}}{\_}$

$9 \cdot 5 \cdot 10 \cdot 5 = 2250$

5. $\underbrace{\_\ \_\ \_\ \_}_{\text{Girls}}\ \underbrace{\_\ \_\ \_\ \_}_{\text{Boys}}$

$(4 \cdot 3 \cdot 2 \cdot 1) \cdot (4 \cdot 3 \cdot 2 \cdot 1) = 576$

6a) $\frac{5!}{2!3!} = 10$

b) $\frac{5!}{3!2!} = 10$

c) $\frac{10!}{4!6!} = 210$

d) $\frac{10!}{6!4!} = 210$

e) $\frac{52!}{47!5!} = 2598960$

f) $\frac{52!}{5!47!} = 2598960$

7. In pairs, when the numbers in the bottom add up to the top number of one combination, it is the same answer

a) $\binom{100}{4}$

b) $\binom{250}{3}$

c) $\binom{n}{n-r}$

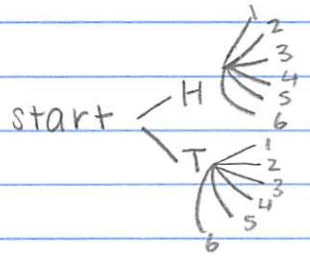
8a) $C_{10,4} = 210$

b) $P_{10,4} = 5040$

SUN
12/14/13

5-3 #2,4,9-16,18,20,22

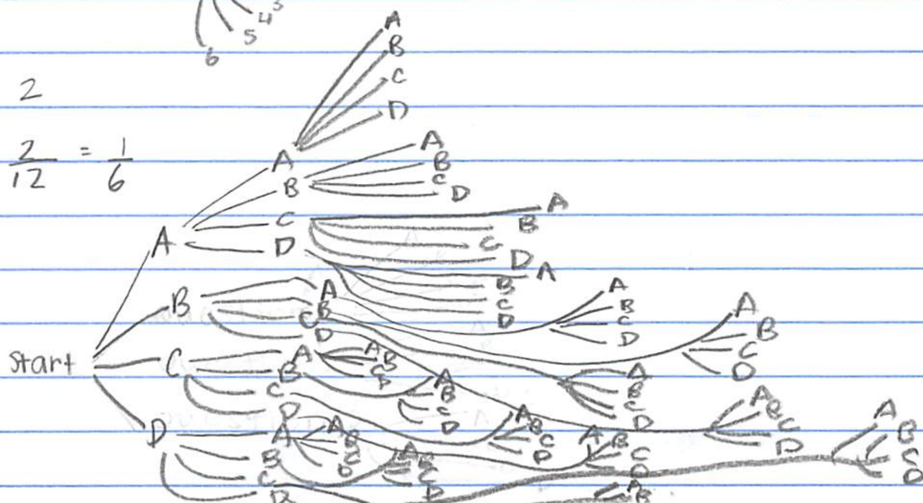
2 a)



b) 2

c) $\frac{2}{12} = \frac{1}{6}$

4 a)



b) P(all correct) = P(Q1 Right) · P(Q2 Right) · P(Q3 Right)

$= \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

$= \frac{1}{64} = 0.0156$

9. $P_{5,2} = \frac{5!}{3!} = 20$

10. $P_{8,3} = \frac{8!}{5!} = 336$

11. $P_{7,7} = \frac{7!}{0!} = 5040$

12. $P_{9,9} = \frac{9!}{0!} = 362880$

13. $C_{5,2} = \frac{5!}{3!2!} = 10$

14. $C_{8,3} = \frac{8!}{3!5!} = 56$

15. $C_{7,7} = \frac{7!}{0!7!} = 1$

16. $C_{8,8} = \frac{8!}{0!8!} = 1$

18. $C_{10,3} = \frac{10!}{3!7!} = 120$

20. a) $C_{12,5} = \frac{12!}{7!5!} = 792$

b) $\frac{1}{792} = 0.001263$

c) $C_{7,5} = 21$

$C_{12,5} = 792$

P(sylvia gets lucky) $= \frac{21}{792} = 0.02652$

22 a) $C_{42,6} = 5245786$

b) $\frac{1}{5245786}$

c) $\frac{10}{5245786}$

Cathy Sun
12/14/13

5 RVW # 1-3, 5, 6, 8, 10, 12, 13

1. P(woman asked for a raise) = 0.24
P(woman gets raise | asked) = 0.45
P(ask and received) = P(ask) · P(received)
= 0.24 × 0.45 = 0.108

2. P(man asked) = 0.20
P(man received | asked) = 0.59
P(man ask and received) = 0.20 × 0.59 = 0.118

3a) No, drawing the first card affects probability of second card
b) $\frac{13}{52} \cdot \frac{13}{52} = 0.0625$
c) $\frac{13}{52} \cdot \frac{12}{51} = 0.05882$

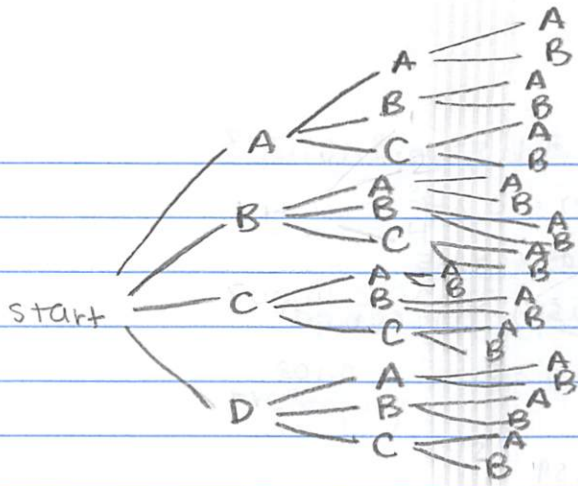
5a) Drop a thumbtack 100 times and record how many times it lands on flat side down to form probability.
b) Land flat side down or land pointy side down
c) P(flat side down) = $\frac{340}{500}$ = 0.68
P(pointy side down) = $\frac{160}{500}$ = 0.32

6a) $P(N) = \frac{470}{1000} = 0.47$
$P(M) = \frac{390}{1000} = 0.39$
$P(S) = \frac{140}{1000} = 0.14$
b) $P(N \mid W) = \frac{420}{500} = 0.84$
$P(S \mid W) = \frac{20}{500} = 0.04$
c) $P(N \mid A) = \frac{50}{500} = 0.10$
$P(S \mid A) = \frac{120}{500} = 0.24$
d) P(N and W) = $\frac{420}{1000}$ = 0.420
P(M and W) = $\frac{60}{1000}$ = 0.06
e) Yes, can't have both reactions, no overlaps
f) No, the probability of no reaction is affected by how long it takes to wash off oil

8. P(101 and 102) = P(101) × P(102)
= 0.77 × 0.90 = 0.693

10a) $P_{7,2} = \frac{7!}{5!} = 42$ d) $C_{4,4} = \frac{4!}{0!4!} = 1$
b) $C_{7,2} = \frac{7!}{2!5!} = 21$
c) $P_{3,3} = \frac{3!}{0!} = 6$

12.



13. (4)(4)(4)(4)(4) = 1024 Answer sequences

$\frac{1}{1024}$ = 0.0009766 of getting all five answers correct

Combinations can be used to answer probability questions as long as we remember that for equally likely outcomes, the probability of an event is the ratio of the number of outcomes in the event to the number of outcomes in the sample space.

EXAMPLE 6

Find the probability that a committee of 10 people chosen from an organization consisting of 40 doctors and 35 dentists will include 3 doctors and 7 dentists.

$$P = \frac{\text{number of ways to choose 3 doctors and 7 dentists}}{\text{number of ways to choose 10 people from 75}}$$

$$= \frac{\binom{40}{3}\binom{35}{7}}{\binom{75}{10}}$$

$$\approx .0801$$

Review Exercises

FREE-RESPONSE QUESTIONS

Open-Ended Questions

1. If a young man owns 5 pairs of pants, 7 shirts, and 4 pairs of shoes, how many outfits can he assemble that consist of 1 pair of pants, 1 shirt, and 1 pair of shoes?

2. If an automobile license plate must consist of three letters followed by three single-digit numbers, how many different license plates are possible? Remember that numbers and letters could be duplicated.

3. If a combination lock has a three-number combination and the wheel on the lock allows any number from 0 to 40, how many different combinations are possible? (Assume that numbers can repeat.)

4. How many four-digit numbers have both the hundreds and units digits even?

5. How many ways can we arrange a row of eight seats consisting of four girls followed by four boys?

6. Calculate each and verify with your calculator.
   a. $\binom{5}{2}$
   b. $\binom{5}{3}$
   c. $\binom{10}{4}$
   d. $\binom{10}{6}$
   e. $\binom{52}{5}$
   f. $\binom{52}{47}$

7. Do you notice the patterns of the pairs in question 6? Equate each combination listed below to an equivalent combination form.
   a. $\binom{100}{96} =$
   b. $\binom{250}{247} =$
   c. $\binom{n}{r} =$

8. a. How many ways can a committee of 4 people be chosen from a club of 10 members?
   b. How many ways can a president, vice-president, secretary, and treasurer be chosen from a club of 10 members?

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EXAMPLE 3

Calculate ${}_5C_2$ directly and by finding ${}_5P_2$ first.

Solution

a. directly

$$
\begin{aligned}
{}_5C_2 &= \frac{5!}{2!(5-2)!}\\
&= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)}\\
&= 10
\end{aligned}
$$

b. by finding ${}_5P_2$ first

$$
\begin{aligned}
{}_5C_2 &= \frac{{}_5P_2}{2!}\\
&= \frac{\frac{5!}{(5-2)!}}{2!}\\
&= \frac{20}{2}\\
&= 10
\end{aligned}
$$

EXAMPLE 4

How many different possible hands of 5 cards each can be dealt from a standard deck of 52 cards?

Solution

The order of the 5 cards does not matter so we use combinations.

$$
{}_{52}C_5 = \binom{52}{5} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2{,}598{,}960
$$

Technology Note: Many scientific calculators have the capability of calculating permutations directly. The TI-83/84 calculators have the command ***nPr*** on the *MATH PRB* menu. Calculation of combinations can be accomplished by formula or by using the calculator: on the TI-83/84 calculators, the command ***nCr*** is found on the *MATH PRB* menu as well. ■

Combinations can be used when dealing with situations that involve more than one choice or event.

EXAMPLE 5

How many ways can a committee of 3 women and 2 men be chosen from an organization comprised of 10 women and 12 men?

Solution

Think of the problem as choosing a committee of 3 women from the 10 women and a committee of 2 men from the 12 men. Therefore, we multiply the number of ways that each can occur:

$$
\binom{10}{3}\binom{12}{2} = \left(\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}\right)\left(\frac{12 \cdot 11}{2 \cdot 1}\right) = 7{,}920
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&= \frac{20}{2} \\
&= 10
\end{aligned}
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Permutations

A permutation of a set of objects is an **arrangement** of some or all of the objects in a specified order: the arrangements of r objects from a set of n objects are called the **permutations** of n objects taken r at a time.

For permutations, the order of the objects is of critical importance. For example, ABC is a different permutation from BAC.

Permutations can be calculated either by applying the Fundamental Counting Principle or by using a special formula. The formula for the number of permutations of n objects taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}, \text{ where } n \geq r.$$

Note that for n objects taken n at a time, $_nP_n = \frac{n!}{0!} = n!$ since $0! = 1$.

EXAMPLE 2

How many ways can you arrange 4 out of 7 books on a shelf?

Solution

a. by the Fundamental Counting Principle

Position	1	2	3	4
Number of Choices	7	6	5	4

$7 \times 6 \times 5 \times 4 = 840$

b. by formula

$$_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$$

You can see why we don't want to have to list all of these possibilities!

Combinations

A combination of a set of objects is a selection of certain of these objects with no regard to order. The sets of r objects from a collection of n objects for which the order of the objects selected is of no importance are called the **combinations** of n objects taken r at a time. The formula for combinations, or "n choose r," is

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}, \text{ where } n \geq r.$$

You should note that this is very similar to the formula for permutations. In fact,

$$_nC_r = \binom{n}{r} = \frac{_nP_r}{r!}$$

Note that $_nC_n = 1$.

By examining the formula above, you can see that the number of combinations is always less than the corresponding number of permutations by a factor of $\frac{1}{r!}$, that is, the reciprocal of the number of arrangements of the r items selected.

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False Positives and Drug Testing

Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs, the false positive rate is 5%, and the false negative rate is 10%. Select one prospective employee at random.

(a) Would it be better to use a tree diagram or a two-way table to summarize this chance process?

(b) What percent of prospective employees will test positive?

(c) What percent of prospective employees who test positive actually use illegal drugs?

Gender and Handedness

Using the random sampler at www.amstat.org/CensusAtSchool, 20 high school students were selected. The gender and handedness of each student is listed in the data table to the right.

Gender	Handed
Female	Right
Male	Right
Female	Right
Male	Right
Male	Right
Male	Right
Female	Right
Female	Left
Male	Left
Female	Right
Female	Right
Female	Left
Male	Right
Male	Right
Male	Right
Female	Right
Male	Right
Male	Right
Female	Right
Female	Right

(a) Create a two-way table to summarize these data. How is a two-way table different than a data table?

	F	M	
R	8	9	17
L	2	1	3
	10	10	

Choose one of these students at random.

(b) What is the probability that the student is female and right-handed?

$$\frac{8}{20} = \frac{2}{5}$$

(c) What is the probability that the student is female or right-handed?

$$\frac{10}{20} + \frac{17}{20} - \frac{8}{20} = \frac{19}{20}$$

(d) Given that the student is female, what is the probability that she is right-handed?

$$\frac{8}{17}$$

(e) Are "selecting a female" and "selecting a right-hander" independent events? Justify.

$$P(F) = P(F \mid R)$$

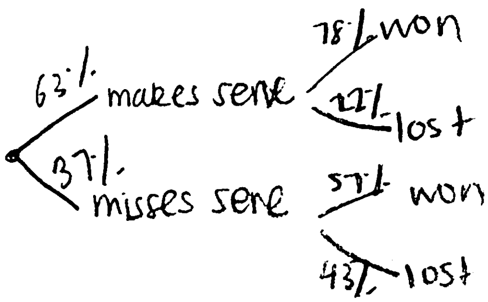
$$\frac{10}{20} \neq \frac{8}{17}$$

not independent

Tree Diagrams, the General Multiplication Rule, and Independence (S-CP.1, 2, 8)

Serve It Up!
Tennis great Roger Federer made 63% of his first serves in the 2011 season. When Federer made his first serve, he won 78% of the points. When Federer missed his first serve and had to serve again, he won only 57% of the points. Suppose we randomly choose a point on which Federer served.

(a) Display this chance process with a tree diagram.



(b) What is the probability that Federer makes his first serve and wins the point?

$0.63 \times 0.78 = 0.4914$

(c) What is the General Multiplication Rule? What if the two events are independent?

P(A and B) = P(A) P(B | A)

P(A and B) = P(A) P(B) if independent

(d) What is the probability that Federer wins the point?

$(0.63 \times 0.78) + (0.37 \times 0.57) = 0.7023$

(e) Given that Federer won the point, what is the probability that he missed his first serve?

$0.7023 \times 0.37 = 0.2599$

Two-Way Tables and the General Addition Rule (S-CP.1, 4, 7)

Free Tacos!
In 2012, fans at Arizona Diamondbacks home games would win 3 free tacos from Taco Bell if the Diamondbacks scored 6 or more runs. In the 2012 season, the Diamondbacks won 41 of their 81 home games and gave away free tacos in 30 of their 81 home games. In 26 of the games, the Diamondbacks won and gave away free tacos. Let W = win and T = free tacos. Choose a Diamondbacks home game at random.

(a) Summarize these data in a two-way table.

	W	L	Tot
T	26	4	30
NT	15	36	51
Tot	41	40	81

(b) What is the probability that the D-backs win?

$\frac{41}{81}$

(c) What is the probability that there are free tacos?

$\frac{30}{81}$

(d) What is the probability that the D-backs win and there are free tacos?

$\frac{26}{81}$

(e) What is the probability that the D-backs win or there are free tacos?

$\frac{41}{81} + \frac{30}{81} - \frac{26}{81} = \frac{45}{81}$

(f) What is the General Addition Rule?

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Conditional probability and independence (S-CP.3–6)

More Tacos!

	Win	Loss	Total
Tacos!	26	4	30
No tacos	15	36	51
Total	41	40	81

(g) What is the probability that there are free tacos, given that the D-backs won the game?

$\frac{26}{41}$

(h) What is the probability that the D-backs win the game, given that there were free tacos?

$\frac{26}{30}$

(i) What is a conditional probability? What notation do we use? Is there a formula?

conditional probability is the event of something happening given another condition

P(B|A) = probability of B given A

P(A or B) = P(A) + P(B) - P(A and B)

P(A and B) = P(A) P(B|A)

(j) When are two events independent? Are the events "D-backs win" and "Free tacos" independent? Justify.

probabilities aren't affected by other event happening

P(D backs ~~win~~) = P(win | tacos)

$\frac{41}{81} \neq \frac{26}{30}$

not independent

Cathy
Sun
+3ec

Conditional Probability and Independence in the Common Core

CMC Annual Conference
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**Materials adapted from *Statistical Reasoning in Sports*, by Josh Tabor and Christine Franklin, *The Practice of Statistics*, 5th edition, by Starnes, Tabor, Yates, and Moore, and *The Annotated Teacher's Edition* for *TPS* 5e, by Josh Tabor. All are published by W.H. Freeman.

From the *Common Core State Standards*

Conditional Probability and the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

(+) 8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

(+) 9. Use permutations and combinations to compute probabilities of compound events and solve problems.

5 Rvw??

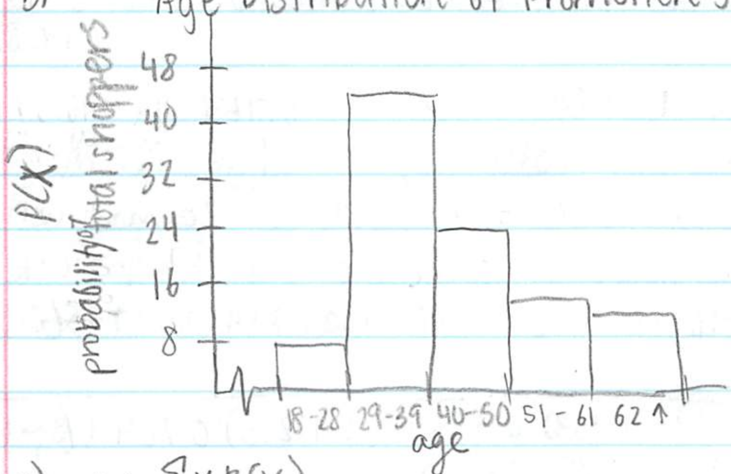
10

Good

cathy sun

6.1 #1-4,7,8

1. a) discrete
   b) continuous
   c) continuous
   d) discrete
   e) continuous
2. a) continuous
   b) discrete
   c) ~~discrete~~
   d) continuous
   e) discrete
3. a) Yes, probabilities add up to 1
   b) NO, probabilities add up to more than 1
4. a) Yes, percentages (probabilities) add up to 1
   b) Age Distribution of Promotion Shoppers



Ana

center: 42.58 - mean

shape: skewed right

spread: $CV = \frac{12.00}{42.58} \times 100 = 28.18\%$, mod low

outliers: none

c) $\mu = \Sigma x P(x)$

$= (23 \times 0.07) + (34 \times 0.44) + (45 \times 0.24) + (56 \times 0.14) + (67 \times 0.11)$

$= 42.58$ years old

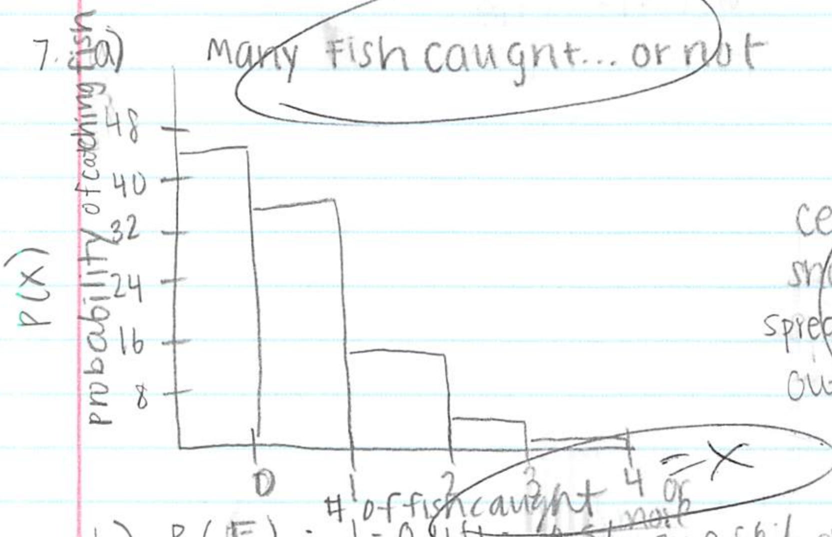
d) $\sigma = \sqrt{\Sigma (x-\mu)^2 P(x)}$

$= \sqrt{(23-42.58)^2(0.07) + (34-42.58)^2(0.34) + (45-42.58)^2(0.24) + (56-42.58)^2(0.14) + (67-42.58)^2(0.11)}$

$= 12.00$

oops!

7. a) Many fish caught... or not



Ana

center: 0.82 - mean

shape: skewed right

spread = $CV = \frac{0.8987}{0.82} = 110\%$ - wild

outliers: 4 or more fish caught

b) $P(F) = 1 - 0.44 = 0.56$ ∃x a 56% chance that 1 or more fish are caught

c) $P(F) = 1 - 0.44 - 0.36 = 0.20$ ∃x a 20% chance of catching 2 or more fish.

d) $\mu = \sum x\, P(x)$

$= (0 \times 0.44) + (1 \times 0.36) + (2 \times 0.15) + (3 \times 0.04) + (4 \times 0.01) = 0.82$

e) $\sigma = \sqrt{(0-0.82)^2 0.44 + (1-0.82)^2 0.36 + (2-0.82)^2 0.15 + (3-0.82)^2 0.04 + (4-0.82)^2 0.01}$

$= 0.8987$

8. a) P(1 or more repeat) = 1 − P(0) = 1 − 0.237 = 0.763 ∃x a 76.3% chance that at least 1 repeat offenders.
is 1 − probability of none of parolees repeat offenses

b) ~~1 −~~ 0.237 − 0.396 = 0.367 ∃x a 36.7% chance that 2 or more will be repeat offenders

c) $P(X \geq 4) = P(4) + P(5)$
0.015 + 0.001 = 0.016 ∃x a 1.6% chance that at least 4 go back to prison

d) $\mu = (0 \times 0.237) + (1 \times 0.396) + (2 \times 0.264) + (3 \times 0.088) + 4(0.015) + 5(0.001) =$

$= 1.253$

e) $\sigma = \sqrt{(0-1.253)^2 0.237 + (1-1.253)^2 0.396 + (2-1.253)^2 0.264 + (3-1.253)^2 0.088 + (4-1.253)^2 0.015 + (5-1.253)^2 0.001}$

= ~~0.9626~~

X	P(X)	X P(X)	$(X-\mu)^2$	$(X-\mu)^2 P(X)$
0	0.237	0	$(-1.25)^2 = 1.5625$	0.3703
1	0.396	0.396	$(-.25)^2 = 0.0625$	0.02475
2	0.264	0.528	$.75^2 = 0.5625$	0.1485
3	0.088	0.264	$1.75^2 = 3.0625$	0.2695
4	0.015	0.060	$2.75^2 = 7.5625$	0.1134
5	0.001	0.005	$3.75^2 = 14.0625$	0.01404

$\sqrt{0.9407} = 0.9699$

CATHY SUN

Mr. Smith situation

Mr. Smith Econ Quiz

10 T/F Questions

P(I get B or above)

S = question correct

F = wrong answer

$n = 10$

$p = 0.50$

$q = 0.50$

$P(r \geq 8) = P(r=8) + P(r=9) + P(r=10)$

$= 0.044 + 0.010 + 0.001$

$= 0.055$

∃ a 5.5% chance that I get a B or above.

6.2 # 1-4, 6, 10-12

11.a) A trial is whether marketing personnel are introverts or extroverts.

S = extroverts, F = introverts, $n = 15$, $p = 0.75$, $q = 0.25$

$P(r \geq 10) = 0.165 + 0.225 + 0.225 + 0.156 + 0.067 + 0.013 = 0.851$

∃x a 85.1% chance that at least 10 are extroverts.

$P(r \geq 5) = 1 - P(r \leq 4) = 1$

∃x an almost 100% chance that at least 5 are extroverts.

$P(r = 15) = C_{15,15}(0.75)^{15}(0.25)^{0} = 0.0134$

∃x a 1.34% chance that all are extroverts.

b) A trial is whether a computer programmer is introverted or extroverted?

S: introvert, F: extrovert, $n = 5$, $p = 0.60$, $q = 0.40$

$P(r = 0) = C_{5,0}(0.60)^{0}(0.40)^{5} = 0.0103$

∃x a 1.03% chance that none is an introvert.

$P(r \geq 3) = 0.346 + 0.259 + 0.078 = 0.683$

∃x a 68.3% chance that 3 or more are introverts.

$P(r = 5) = C_{5,5}(0.60)^{5}(0.40)^{0} = 0.0778$

∃x a 7.78% chance that all are introverts.

12) A trial is whether men would welcome women taking initiative.

S = yes, F = no, $n = 20$, $p = 0.70$, $q = 0.30$

a) $P(r \geq 18) = 0.028 + 0.007 + 0.001 = 0.036$

∃x a 3.6% that at least 18 men will welcome women taking iniative.

b) $P(r < 3) = 0$

∃x almost 0% chance that fewer than 3 men will say yes.

c) $P(r = 0) = C_{20,0}(0.70)^{0}(0.30)^{20} = 3.49 \times 10^{-11}$

∃x almot 0% chance that none say yes

d) $P(r \leq 4) = 0$

∃x almost 0% chance that at least 5 men say no.