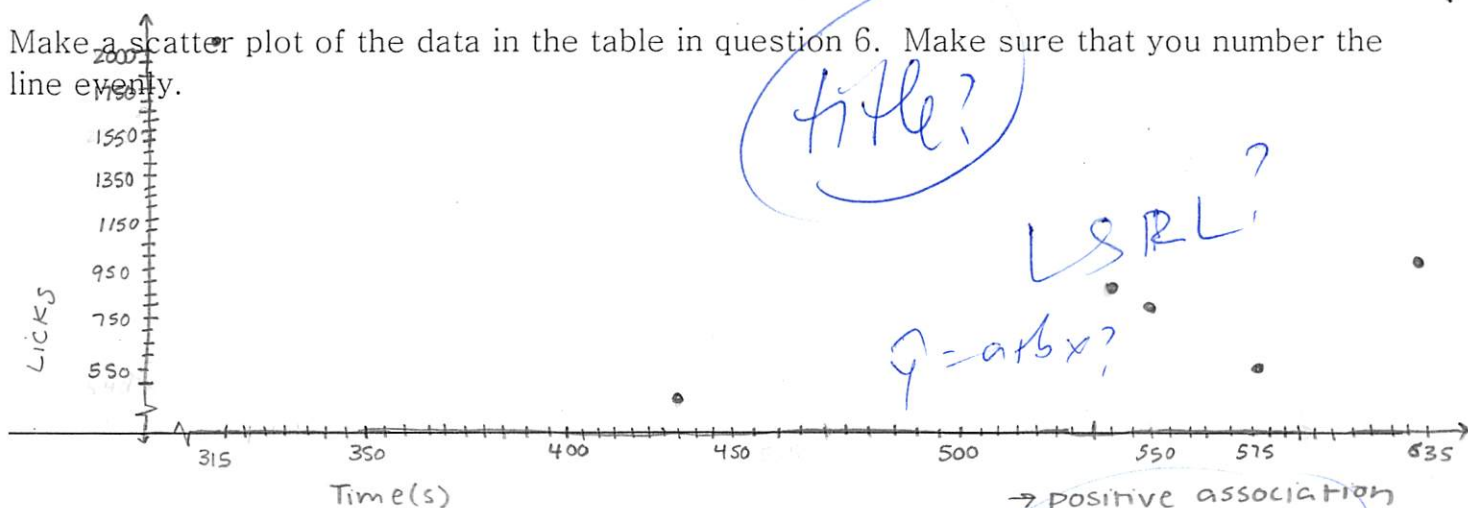
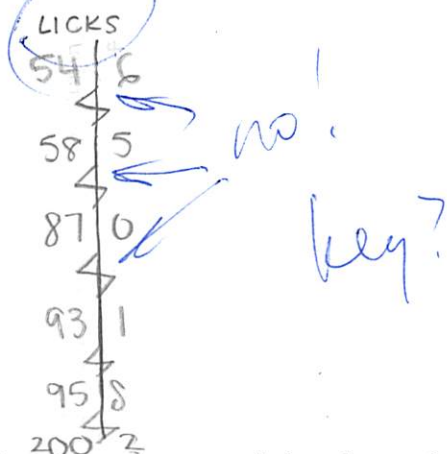


7. Make a scatter plot of the data in the table in question 6. Make sure that you number the line evenly.



8. Make a stem-and-leaf plot of the # of licks data.



9. Describe the appearance of the data. Include shape, gaps, outliers, skewness, etc.

Data is skewed high. outliers are (315, 2002). Gaps between Times 435 and 537. Center? Spread?

10. Get a calculator and record these numbers in a list. Then make a histogram of the data. Does the histogram have the same appearance as the other two plots? Yes No

If no, suggest a way to remedy the problem. Bigger sample size to prevent skewing.

Be ready to discuss your remedy with the class.

20 40 60 85 100/120/140/150/160  
180/200/230/260/280/300/320/340  
360  
380  
400

1. Tootsie Roll Industries is aware of three scientific studies that have been conducted to determine how many licks it takes to reach the center of a Tootsie Pop.
  - A group of engineering students from Perdue University recorded that a licking machine, modeled after a human tongue, took an average of 364 licks to get to the center. They tried the same licking test on 20 volunteers, and found that the average licks to the center were 252.
  - A chemical engineering doctorate student at the University of Michigan recorded that his licking machine required an average of 411 licks.
  - A group of students at Swarthmore School did an in-school experiment using humans, and determined that it took an average of 144 licks to get to the center.
2. How many licks do you think that it will take to get to the chocolate center of the pop without crunching it? about 300 licks
3. When you enjoy a Tootsie Pop, what constitutes a lick? One tongue stroke
4. For this experiment a lick will be defined as one tongue stroke
5. Record below the number of licks as defined above that it takes you to get to the center of your pop. Let 1 tick = 5 licks. DO NOT BITE or CRUNCH IT!!! When you have reached the center, inform your teacher of your number.

931 licks

6. Now record the number of licks it took other students in the class to reach the center of their Tootsie Pops.

Name	Licks	Time (s)	Name	Licks	Time
1.	546	435	2.		
3.	870	550	4.		
5.	2002	315	6.		
7.	685	575	8.		
9.	931	637	10.		
11.	968	537	12.		
13.			14.		
15.			16.		
17.			18.		
19.			20.		

# How Many Licks to the Center of a Tootsie Pop

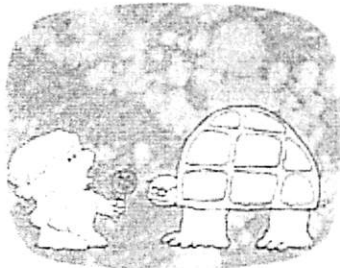
<http://www.tootsie.com/howmany-sb.html>

## Memories ~ Classic TV Spots

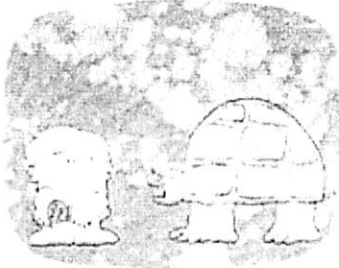


### Tootsie Fable, "How Many Licks"

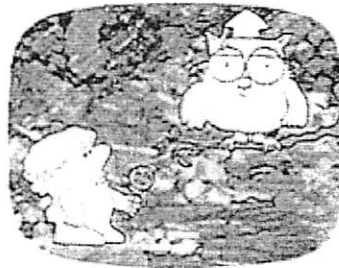
30 Second Commercial



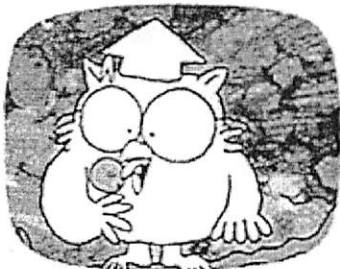
Mr. Turtle, how many licks does it take to get to the Tootsie Roll center of a Tootsie Pop?



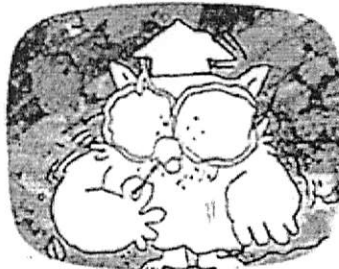
I never made it without biting, ask Mr. Owl.



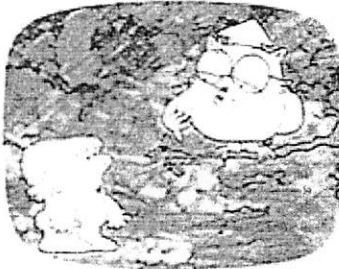
Mr. Owl, how many licks does it take to get to the Tootsie Roll center of a Tootsie Pop?



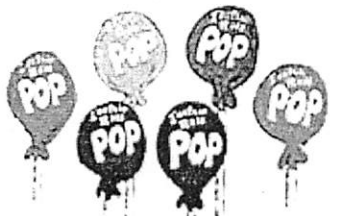
Let's find out. One, two, three.



Crunch



Three



How many licks does it take to get to the Tootsie Roll center of a Tootsie Pop?



Crunch



The world may never know.

cathy sun  
12/5/13

# Birthdays

2 sts	$1 - \frac{364}{365} = 0.002740$
3 sts	$1 - \frac{364}{365} \times \frac{363}{365} = 0.0082$
4	$1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} = 0.0164$
5	$1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \frac{361}{365} = 0.0271$
6	$1 - " \times \frac{360}{365} = 0.0405$
7	$1 - " \times \frac{359}{365} = 0.0562$
8	$1 - " \times \frac{358}{365} = 0.0743$
9	$1 - " \times \frac{357}{365} = 0.0946$
10	$1 - " \times \frac{356}{365} = 0.1169$
11	$1 - " \times \frac{355}{365} = 0.1411$
12	$1 - " \times \frac{354}{365} = 0.1670$
13	$1 - " \times \frac{353}{365} = 0.1944$
14	$1 - " \times \frac{352}{365} = 0.2231$
15	$1 - " \times \frac{351}{365} = 0.2529$
16	$1 - " \times \frac{350}{365} = 0.2836$
17	$1 - " \times \frac{349}{365} = 0.3150$
18	$1 - " \times \frac{348}{365} = 0.3469$
19	$1 - " \times \frac{347}{365} = 0.3791$
20	$1 - " \times \frac{346}{365} = 0.4114$
21	$1 - " \times \frac{345}{365} = 0.4437$
22	$1 - " \times \frac{344}{365} = 0.4757$
<span style="border: 1px solid black;">23</span>	$1 - " \times \frac{343}{365} = 0.5073$
24	$1 - " \times \frac{342}{365} = 0.5383$
25	$1 - " \times \frac{341}{365} = 0.5687$
26	$1 - " \times \frac{340}{365} = 0.5982$
27	$1 - " \times \frac{339}{365} = 0.6269$
28	$1 - " \times \frac{338}{365} = 0.6545$
29	$1 - " \times \frac{337}{365} = 0.6810$
30	$1 - " \times \frac{336}{365} = \span style="border: 1px solid black;">0.7063$

which is better  
than 2:1 odds

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

10-17-19

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10-17-19

10-17-19

10-17-19

10-17-19

10-17-19



# WHAT'S THE CHANCE



## of being invited to 2 Birthday Parties in 1 Day?

Not bad, if you know at least 30 people who have birthday parties every year. Like your class at school. In any group of 30 people, there is a better than 2 to 1 chance that 2 people will have the same birthday.

Mathematicians who study probability theory (and like birthday parties) have proven this to be true.

Try it yourself. Take a poll. Ask people when their birthdays are and stop when you find 2 with the same one. Sometimes it happens right away.

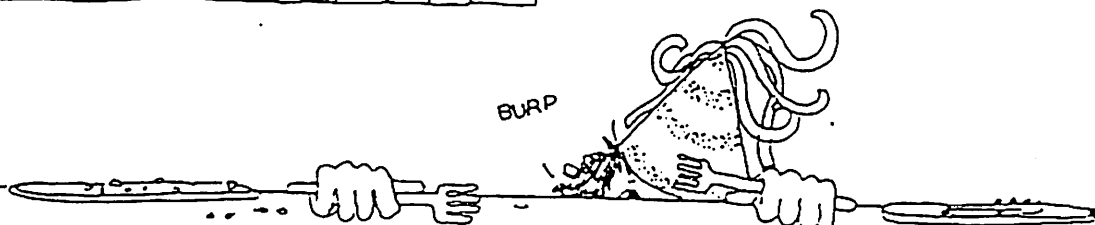


Sometimes it takes a long time.



But if you take the poll *enough times*, the average number of people you have to ask will get very close to 30. If you get sick of asking people when their birthdays are before you get an average of 30, you'll just have to take our word for it.

Are there more birthdays in some months than others? Are the chances that 2 people will have the same birthday better in June than in December?



# BIRTHDAY PROBABILITY

## A Little Theory...

If an event could never happen, its probability is 0.

If an event is certain to happen, its probability is 1.

When tossing 2 coins, 2 things could happen: H or T. The probability of heads  $P(H) = .5$  and the probability of tails  $P(T) = .5$ . Since it is certain that either heads or tails will happen,  $P(H) + P(T) = 1$ . Another way of saying the same thing is  $1 - P(H) = P(T)$  or  $1 - .5 = .5$ .

Another example: If the probability of rain is .7, then the probability of no rain is  $1 - .7$  or .3.

## The Problem...

Given a class of students, what is the probability that 2 or more of them have their birthday on the same day? (Just the day and month, not the year)

## The Solution...

Figure out the probability of this happening with only 2 students in the class: Since there are 365 days (don't worry about leap year) in the year, the first person could have any date. Then the probability that the second person has the same birthday is as follows:

Find out the probability that their birthdays aren't the same, and subtract from 1.  $P(\text{same}) = 1 - P(\text{not the same})$ . Since there are 364 days in the year that don't match,  $P(\text{not the same}) = 364/365$ . So  $P(\text{the same}) = 1 - 364/365$  or about .00273.

Try the same strategy with a class of 3 students.

$P(\text{the same}) = 1 - 364/365 \times 363/365$  or about .00804

For 4 students:  $1 - 364/365 \times 363/365 \times 362/365$  or about <sup>.0164</sup>~~.00814~~

The probability of having the same birthday passes 50% at a class size of about 25 students.

5 sts: 1 - "

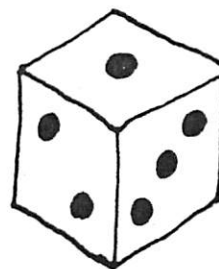
$$\times \frac{361}{365}$$

Find 17 sts

30 sts: show if right or wrong



# PIG



Here is a good game. It depends on knowing a little something about probability as well as not being too much of a pig.

You need 2 dice, a friend, and a paper and pencil (unless you are terrific at adding numbers in your head).

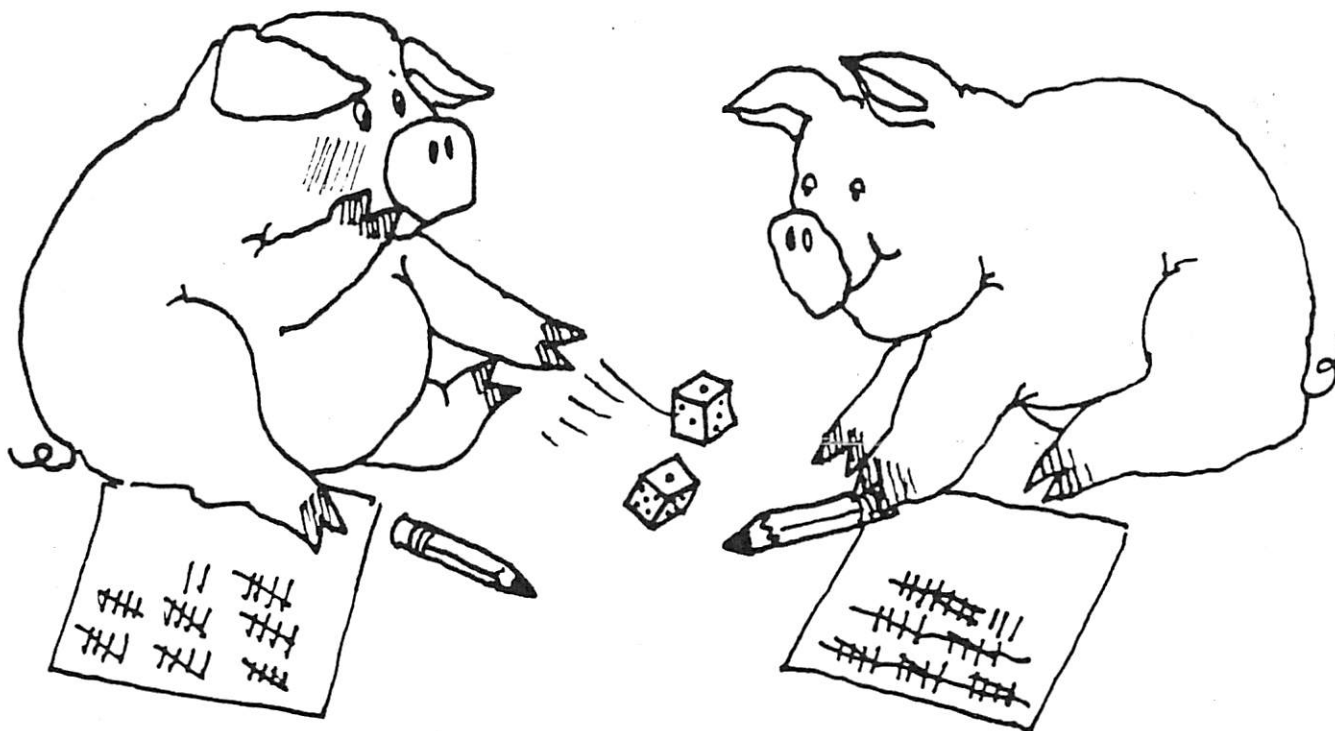
You roll the dice and add up what they say. The idea is to get to 100. You don't have to take turns. You keep rolling as long as you want. BUT:

If a 1 comes up on 1 of the dice, you lose your count for that turn.

If a 1 comes up on both dice, your total goes back to 0. (Even if you were at 98!) And anytime you throw a 1, you lose your turn.

It helps a lot to know how to add. But it helps even more if you can predict how often 1's will come up. What is the probability of throwing one 1? What is the probability of throwing snake eyes (two 1's)?

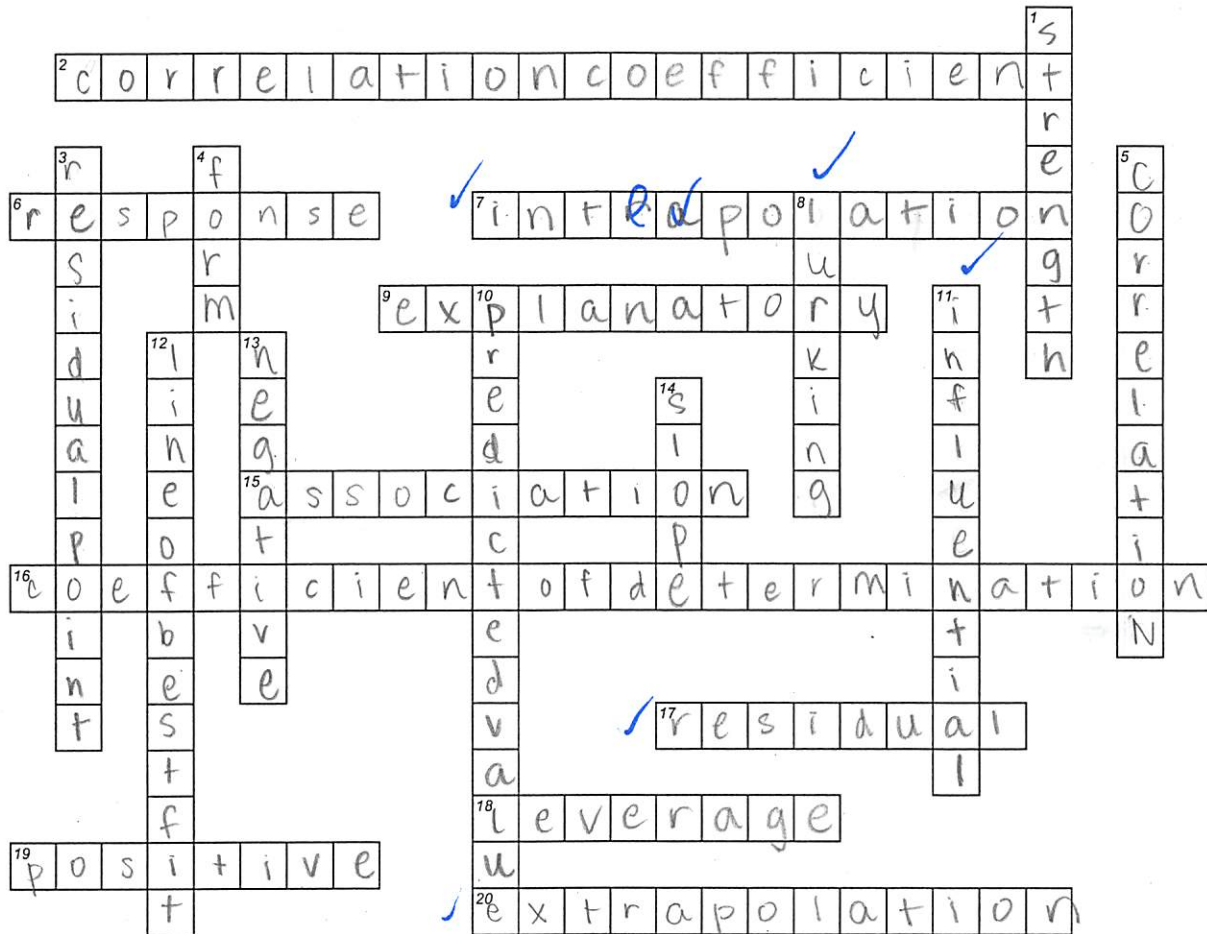
What is a lucky streak? Do you know one when you have it?





# Exploring Relationships Between Variables

## Advanced Placement Statistics



Stats: Modeling the World, Chapters 7-10

### ACROSS

- 2 numerical measure of the direction and strength of a linear association
- 6 variable that you hope to predict or explain
- 7 predicting for values of  $x$  within the ones used to find the linear model equation
- 9 variable that accounts for, explains, predicts, or is otherwise responsible for the  $y$ -variable
- 10 relationship between two quantitative variables
- 11 overall measure of how successful the regression is in linearly relating  $y$  to  $x$
- 12 the difference between the actual data value and the corresponding value predicted by a model
- 13 data points whose  $x$ -values are far from the mean of  $x$  have a high amount of this
- 14 type of association where as one variable increases, so does the other
- 15 predicting for values of  $x$  far from the ones used to find the linear model equation

### DOWN

- 1 general measure of scatter around the underlying relationship between two quantitative variables
- 3 point on the scatterplot representing the mean  $x$ -value and mean  $y$ -value
- 4 shape of a scatterplot
- 5 shows the relationship between two quantitative variables measured on the same cases
- 8 a variable that is not explicitly part of a model but affects the way the variables in the model appear to be related
- 10  $y$ -hat
- 11 point that when omitted, results in a very different regression model
- 12 least squares regression line
- 13 type of association where an increases in one variable generally correspond to decreases in the other
- 14 measures the change in the  $y$ -value per unit change in  $x$ -value

Cathy Sun  
12/3/13

5.1 # 3, 5, 7-10

3. B, D, H - cannot be over 1 or negative

5. out of 10 friends, 1 can wiggle ears, which is 10%.  
since it is a sample, this result could not accurately represent a large population because my group of friends is slightly less adept at ear wiggling.

7. a)  $P(0) = \frac{15}{375} = \frac{1}{25}$

$$P(1) = \frac{71}{375}$$

$$P(2) = \frac{124}{375}$$

$$P(3) = 131/375$$

$$P(4) = 34/375$$

b) yes since the sample adds up to 375 (or all the married couples), the sample space is 0, 1, 2, 3, or 4 <sup>similar</sup> preferences

8. a)  $P(\text{not engaged}) = 1/5$

$$P(< 1 \text{ year dated}) = 240/1000 = 6/25$$

$$P(1 < \text{years} < 2) = 210/1000 = 21/100$$

$$P(> 2 \text{ years}) = 350/1000 = 7/20$$

b) yes since all 1000 couples responded and must pick an answer. The sample space is never engaged, < 1 year, 1 to 2 years, > 2 years

9. a)  $P(6 \text{ - noon}) = 290/966 = 145/483$

$$P(12 \text{ - 6 PM}) = 135/966 = 45/322 = 15/161$$

$$P(6 \text{ PM - midnight}) = 319/966$$

$$P(\text{Midnight - 6 AM}) = 222/966 = 37/161$$

b) yes, all 966 inventors answered survey, sample space is 6 AM - noon,

10. a)  $2430/3000 = 81\%$  chance of germination

b)  $570/3000 = 19\%$  chance of not germinating

c) sample space is germinating or not, yes they add up to 1, and it should be there are only two options, germinating and not germinating, and the probabilities add up to 1

d) No, more likely to germinate

12 noon - 6 PM  
6 PM - 12 midnight  
12 midnight - 6 AM

1. Find the value of  $x$  if  $\log_2 x = 3$ .

\_\_\_\_\_

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\_\_\_\_\_

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10-10-10

0.0000

\_\_\_\_\_

\_\_\_\_\_

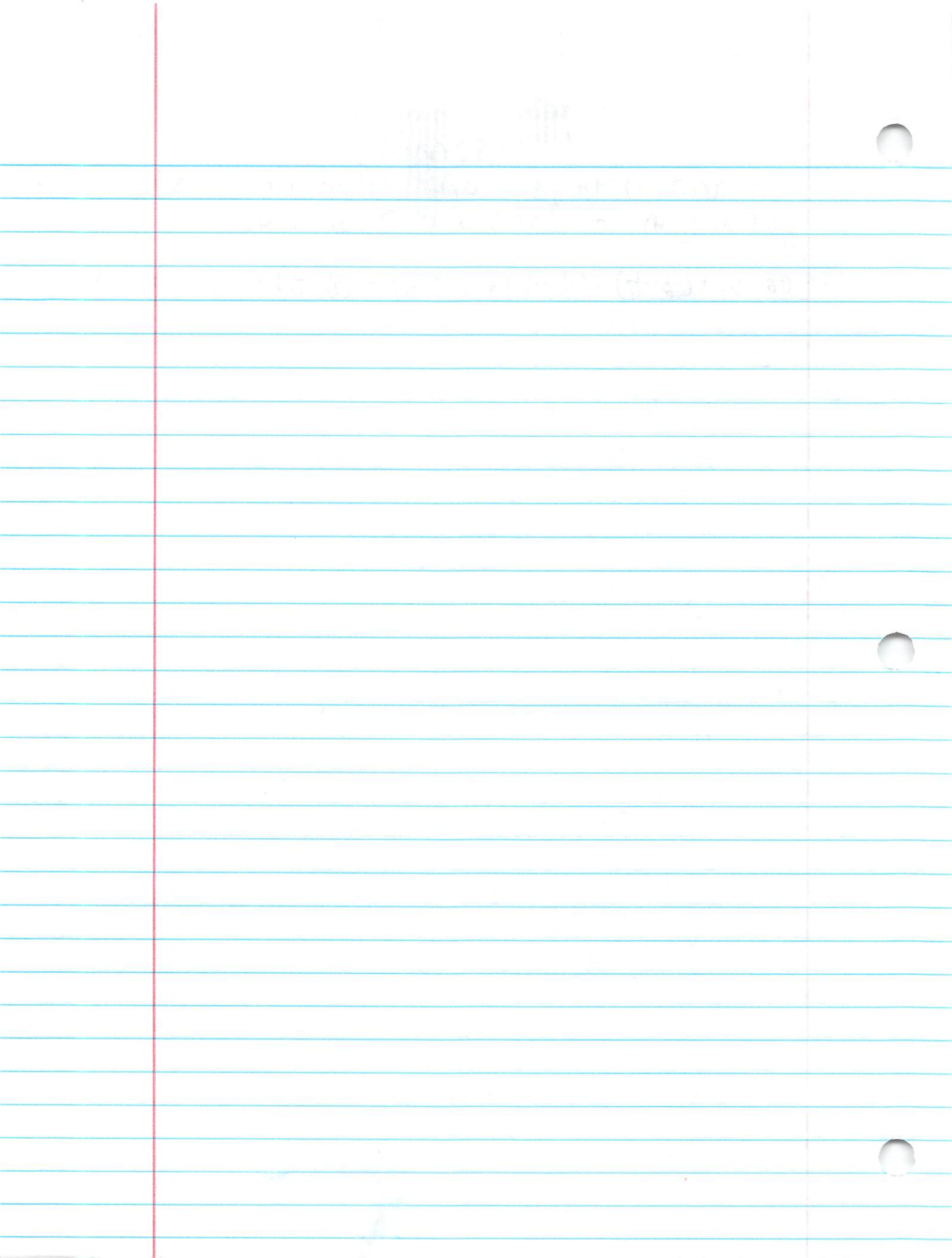
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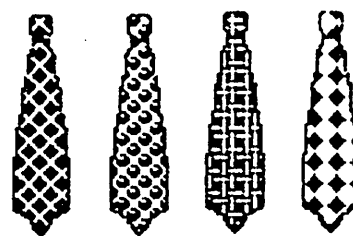








## Mr. Spiffy's Tie Problem - page 2



Week	Tie	Tie	Tie	Tie	Tie
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					

Compute the probability:

$$\frac{\text{number of weeks he wore the same tie more than once}}{\text{total number of weeks}} = \frac{\boxed{\phantom{000}}}{18}$$

Use the data you generated to answer these questions:

What is the probability that he will wear the same tie three times in one week?

What is the probability he will wear a different tie each day of the week?

Another problem the students wanted to figure out was how many days in a row it is likely that he would wear a different tie. To do this they had to devise a different simulation. For this problem they decided to pick out a tie, put it back in the box, choose again until they chose one like one they had picked before. They did this 20 times and then computed the average number of days it took before he wore the same tie twice.

One trial might look something like this: ACD/A 3 days

Do this experiment and record your results below.

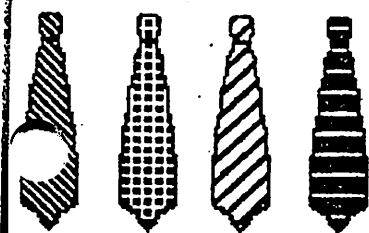
Trial	Result	# of days
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Trial	Result	# of days
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Compute the average:

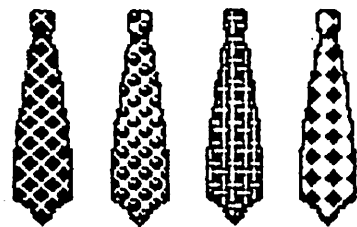
$$\frac{\text{total number of days}}{\text{number of trials}} = \frac{\boxed{\phantom{000}}}{20}$$





# MR. SPIFFY'S TIE PROBLEM

By Crystal Mills

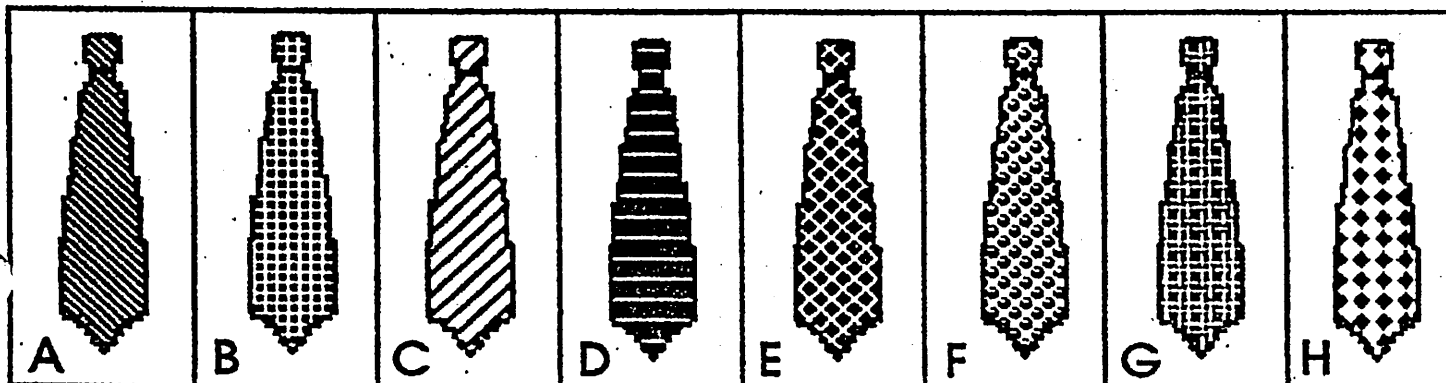


Every morning in his rush to get to school on time, Mr. Spiffy reaches into his closet and pulls out a tie. He doesn't even look at the tie until he is in his car. During the long commute in the slow rush hour traffic, he ties his tie. At night when he gets home, he takes his tie off and puts it back in the closet. The next day he repeats the same ritual.

Mr. Spiffy's students think they have been noticing of late that he often wears the same tie at least twice in one week, but since he picks a tie at random each morning, Mr. Spiffy denies this. His students are persistent and they are determined to prove to him that even though he chooses his ties at random, the probability that he will wear the same tie more than once each week is greater than 50%.

Knowing that he has eight different ties, they design a simulation to figure out the problem. On eight pieces of paper, they draw a tie to represent each of his different ties. All of the "ties" are placed in a box. One is drawn and then put back in the box. This process is repeated five times to represent the five days in a week. Each time a tie is drawn, they record the results in a table like the one on the next page. They continue doing this until they have enough data for 18 weeks. Then they count the number of weeks in which he would have worn the same tie more than once. Next they figure out the probability. Who do you think was right, Mr. Spiffy or his students? Try doing the experiment to find out.

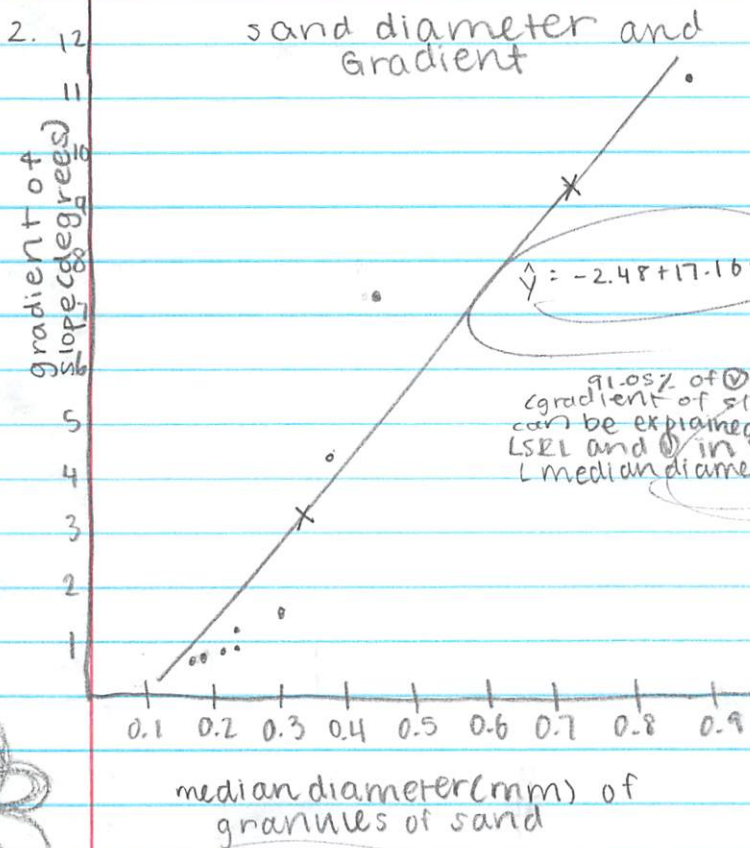
Record your results in the table on the next page.



10

# P162 Beaches

1.  $\bar{x} = 0.33$      $\bar{y} = 3.19$   
 $s = 0.2107$      $s = 3.79$



## ANA

- positive association
- $r = 0.9542$ , high
- as median diameter of granules of sand increase, so does gradient of slope
- (0.85, 11.3) - influential pt
- (0.42, 7.3) - outlier

$\rightarrow \text{COD} = r^2 = 0.9105$

Expect moderately high correlation and good fit b/c previous research showed there to be strong relationship between sand size and slope

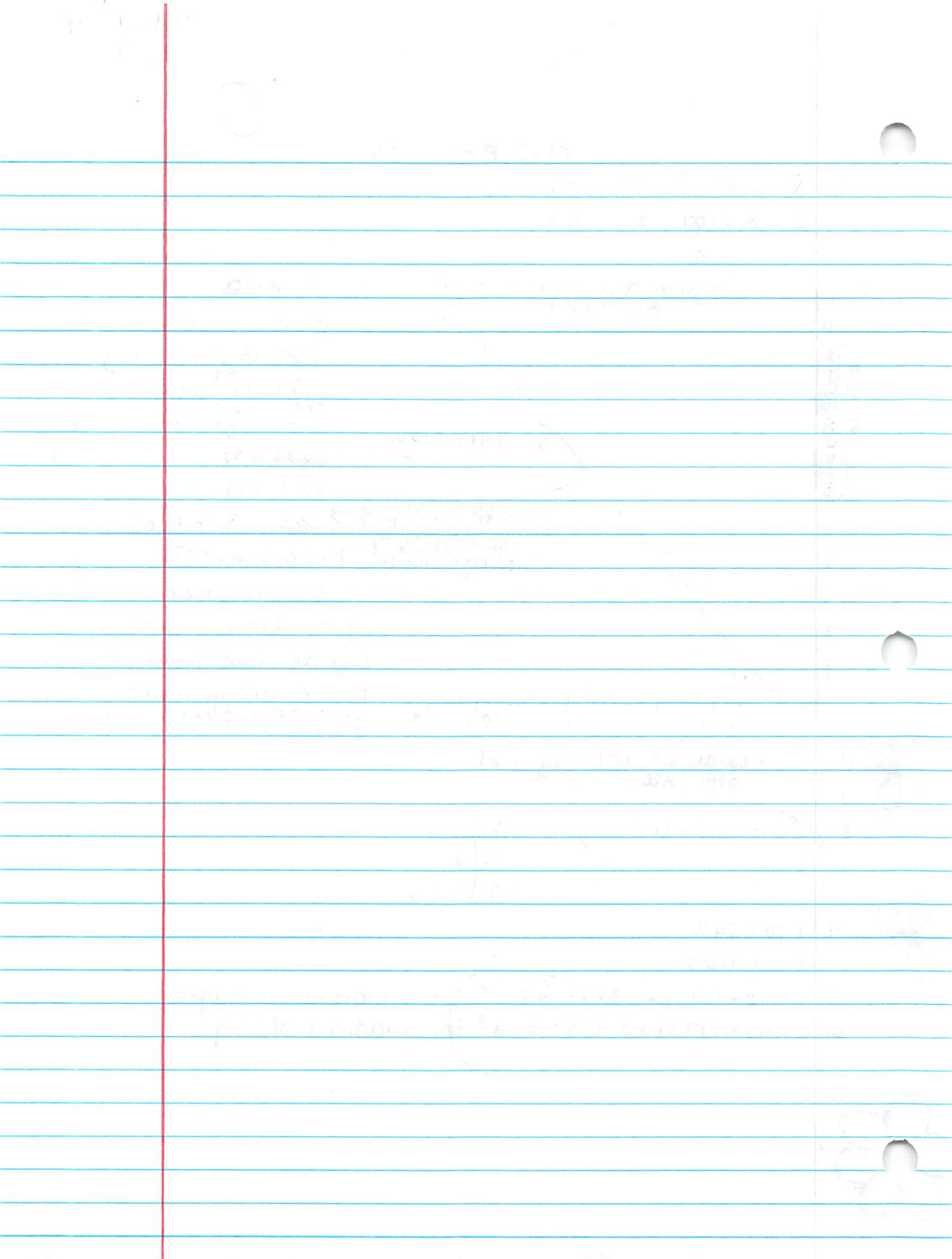
3.  $\hat{y} = -2.48 + 17.16x$

X	Y
0.33	3.19
0.7	9.53

4.  $r = 0.9542$   
 $r^2 = 0.9105$

5.  $-2.48 + (17.16 \times 0.38) = 4.04^\circ$  for gradient of slope

6.  $-2.48 + (17.16 \times 0.45) = 5.24^\circ$  for gradient of slope



Cathy Sun

### Fortune Hunter

P(\$, \$, \$, \$, \$)

↑   ↑   ↑   ↑   ↑  
 $\frac{5}{25}$   $\frac{4}{24}$   $\frac{3}{23}$   $\frac{2}{22}$   $\frac{1}{21}$

$$\frac{5}{25} \cdot \frac{4}{24} \cdot \frac{3}{23} \cdot \frac{2}{22} \cdot \frac{1}{21} = 1.88 \times 10^{-5}$$



1/10/2020

1. The first part of the paper is a review of the literature on the topic of the paper. This is followed by a discussion of the methodology used in the study. The results of the study are then presented, followed by a discussion of the implications of the findings. Finally, the paper concludes with a summary of the main points and a list of references.

2. The second part of the paper is a discussion of the implications of the findings. This is followed by a summary of the main points and a list of references.

# 5.3 STUDY GUIDE

SUN

12/12/13

• Begin by reading pages 193 →

• Answer the following questions about tree diagrams, permutations and combinations.

① When outcomes are equally likely, how do we find the  $P(A)$ ?

$$P(A) = \frac{\# \text{ of outcomes favorable to event } A}{\# \text{ of outcomes in sample space}}$$

② Identify the limitation to the approach above.  
not easy to use when there are many outcomes or events are complicated

③ A display of outcomes made of a series of activities is:  
a) laundry list b) world series c) tree diagram

④ Although most of us haven't registered for University coursework, what is the reality of Figure 5-6, the tree diagram of course schedule options, not very realistic because times could conflict.

⑤ Compare the tennis match tree diagram (p. 195 pt. c) with the tennis match sample space (p. 195 pt. e).  
same outcomes/sequences, no real difference. tree is more visual.

⑥ Write and calculate eleven factorial.  
 $11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 39916800$

⑦ When does  $0 = 1$ ?  $0! = 1$

⑧ Calculate  $P_{5,2}$ ;  $P_{5,3}$ ;  $P_{5,4}$ ;  $P_{5,5}$ ; What is the meaning of  $P_{5,5}$ ?  
 $P_{5,2} = 20$   $P_{5,3} = 60$   $P_{5,4} = 120$   $P_{5,5} = 120$   
• way of arranging 5 objects into 5 positions when order matters  
• # of ordered combinations

⑨ T/F the order of items is not important with combinations.

⑩ Match.

$\frac{n!}{(n-r)!}$	Combinations
$\frac{n!}{r!(n-r)!}$	Permutations



(11) Compare Combinations and Permutations.  
Permutations involve groupings and order, but combinations are not concerned about order.

(12) In the Political Science books example  
why do we compute the # of combinations?  
It doesn't matter for the order of the books  
in which you read

(13)  $C_{n,r}$  means ?  
N objects taken r at a time

(14)  $C_{10,4} = \frac{10!}{4!6!} = 210$

(15) \* Record button by button steps for TI calculation  
of  $C_{10,4}$ .

- 1) 10 (n)
- 2) Math
- 3) PRB
- 4) 3 - nCr
- 5) 4 (r)
- 6) enter

Cathy Sun  
12/8/13

10

## SOCKS

P(matching socks) 5 Black  
P(BLK and BLK) or P(Blue & Blue) 8 Blue  
or P(R & R) or P(Gry & Gry) or 1 Brown  
P(Gre & Gre) 3 Red

9 Gray  
2 Green

28 Total

$$P(\text{BLK}) \cdot P(\text{BLK}|\text{BLK}) \\ \frac{5}{28} \cdot \frac{4}{27} \\ +$$

$$P(\text{Blue}) \cdot P(\text{Blue}|\text{Blue}) \\ \frac{8}{28} \cdot \frac{7}{27} \\ +$$

$$P(\text{R}) \cdot P(\text{R}|\text{R}) \\ \frac{3}{28} \cdot \frac{2}{27} \\ +$$

$$P(\text{Gry}) \cdot P(\text{Gry}|\text{Gry}) \\ \frac{9}{28} \cdot \frac{8}{27} \\ +$$

$$P(\text{Gre}) \cdot P(\text{Gre}|\text{Gre}) \\ \frac{2}{28} \cdot \frac{1}{27}$$

= 0.2063 probability of matching socks

4305

2012

2012

2012

2012

2012

2012

2012

2012

2012

2012

2012

2012

+

2012

2012

2012

2012

2012

2012

C, P, R

3.2.1

$$\left\{ \begin{array}{ccc} R & P & C \\ R & C & P \\ C & P & R \\ C & R & P \\ P & R & C \\ P & C & R \end{array} \right\}$$

$$P(G.C) = \frac{1}{6}$$

Expected # of guessing correctly

$\frac{1}{6}$  of 28 = about 4-5

2, 3,

6, 7

not unlikely either

13/28 = maybe difference in taste

independent

dependent

$$P(6, 6)$$

$$P(K, K)$$

compound events

↳ in succession

$$P(A \text{ and } B) =$$

$$P(A \text{ and } B) =$$

$$P(A) \cdot P(B)$$

$$P(A) \cdot P(B|A)$$

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$\frac{4}{57} \cdot \frac{3}{52} = \frac{1}{221} \rightarrow B \text{ given that } A \text{ happens}$$

PIG

P(snake eyes)

P(1 and 1)

$$P(1) \cdot P(1)$$

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$0.02777$$

P(X and 1)

P(1 and 1') and P(1' and 1)

$$P(1) \cdot P(1')$$

$$P(1') \cdot P(1)$$

$$\frac{1}{6} \cdot \frac{5}{6} = \frac{5}{36}$$

$$\frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$$

$$\frac{10}{36} = \frac{5}{18}$$

$$0.27777$$

$$\frac{1}{36} + \frac{10}{36} = \frac{11}{36}$$

11 times, will see a 1

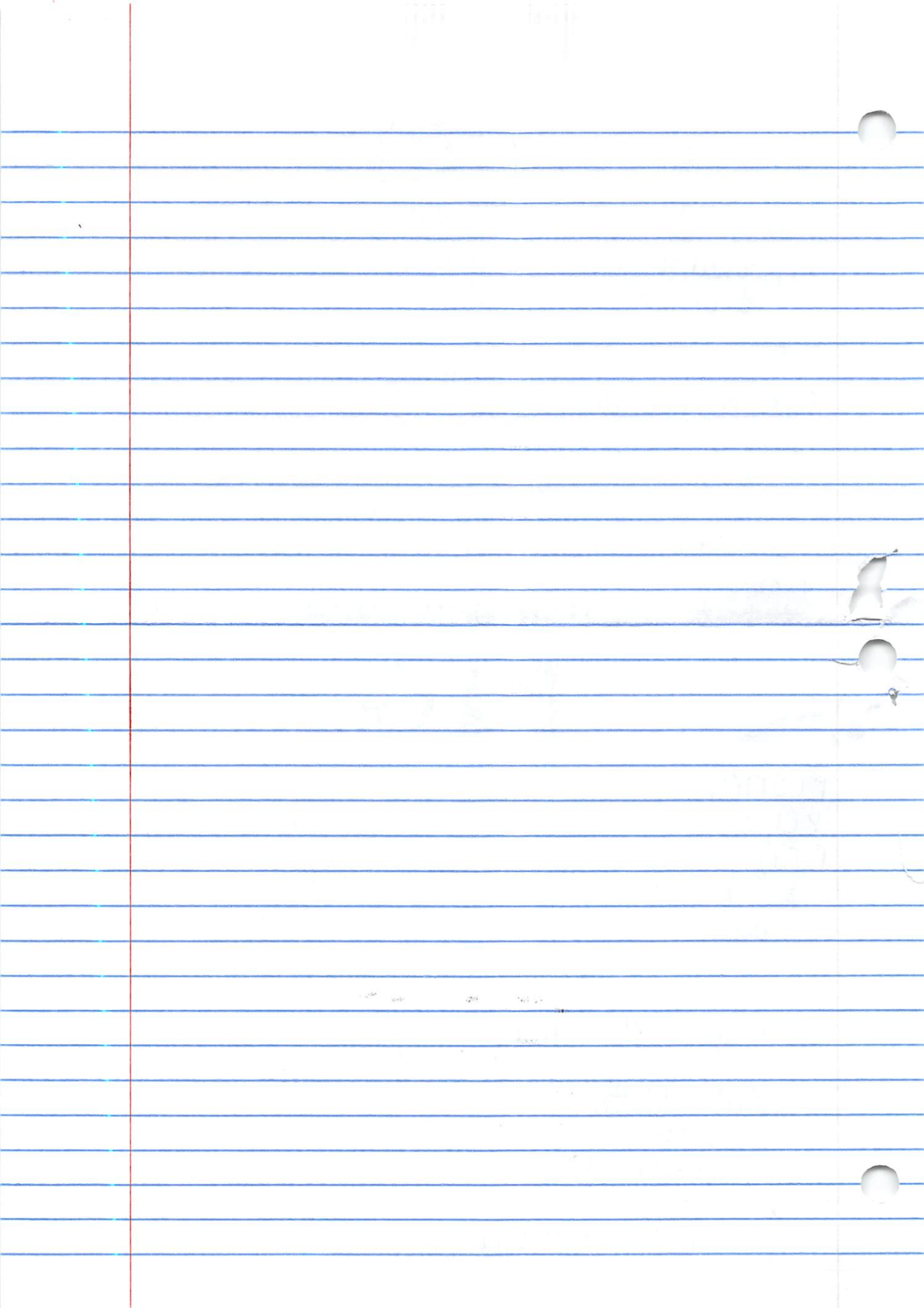
$$\frac{11 \text{ times}}{36 \text{ rolls}} = \frac{1 \text{ time}}{X \text{ rolls}}$$

$$36 = 11X$$

$$X = 3.27 \text{ rolls}$$

> 3 rolls ; not streak





cathy sun  
12/6/13

# PIG

Opponent

me

~~5~~  
~~14~~  
~~5~~  
~~10~~  
16  
~~22~~  
31  
~~40~~  
~~45~~  
~~37~~  
~~44~~  
~~37~~  
~~46~~  
~~56~~  
63  
~~78~~  
~~72~~

~~7~~  
~~15~~  
~~5~~  
17  
~~25~~  
~~32~~  
40  
~~80~~  
60  
~~68~~  
~~78~~  
82  
~~92~~  
~~81~~  
~~92~~  
101







a)  $\frac{291}{2008} = 0.14449$

b)  $\frac{77}{452} = 0.1704$

c)  $\frac{826}{2008} = 0.4114$

d)  $\frac{131}{373} = 0.3512$

e)  $\frac{41}{157} = 0.2611$

f)  $\frac{53}{157} = 0.3376$

g)  $\frac{420}{452} = 0.9292$

h)  $\frac{332}{373} = 0.8901$

i) No, because the probability isn't the same

$$\frac{452}{2008} = \frac{118}{502}$$

$$0.2251 \neq 0.2206$$



1. 10. 10. 2010  
2. 10. 10. 2010  
3. 10. 10. 2010

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