Probability

- Measure of chance
- likeliness of an event occuring
- P(A), prob. of event A "P of A"
- prob found: $\boxed{\frac{f}{n}}$ ($\frac{\text{\# of desired}}{\text{\# of total of results}}$)
- fraction, decimal, %, , $0 \leq P \leq 1$
- certain: 100% chance
- impossible: 0% chance
- need a # and word, depends on context
  - 80+ "highly likely"
    but things like making it alive thru the day wouldn't be high
  - 15- "highly unlikely"
- at least, at most (keep in mind). ($\# \leq$, $\# \geq$)
  anything but a #
  ↳ 100% - percent chance of #
  "complement

- $\mathcal{E}(x) = n \cdot p$

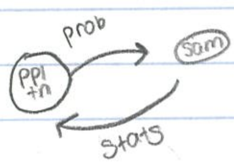
eg: Marbles (3 striped, 2 white, 1 black)

- $P(str) = \frac{\#f}{n} = 3/6 = 50\%$ "moderately likely"
- $P(marble) = \frac{6}{6} = 100\%$ "certain"
- $P(pizza) = \frac{0}{6} = 0\%$ "impossible"

Complement

- P (not the event), anything but a __
  - P(not A), $P(\bar{A})$, $P(A')$
- $P(A) + P(\bar{A}) = 1$
  ↳ $P(\bar{A}) = 1 - P(A)$

Probability vs Statistics



- Stats: Sample is known, draw conclusions based on results
- prob: population is known, you know the entire population
- Stats: sample → population concl'n
- prob: population → sample concl'n

Odds

- favor to unfavorable (not total)
  - (fav : fav', or fav/fav')
- odds of tails on a coin = 1:1
- odds of 6 on a die = 1:5
- u can tell probability by getting total from adding F + F' = T then F/T
  eg 100:1 is $\frac{100}{101}$ odd

Law of Large Numbers

- fallacy of the short run, can't really judge based on just a few trials
- real life approximates theoretical probability in the long run

Sample Space

- S = { the set of all possible outcomes }
- coin toss, S = { heads, tails }
  - 2 coins, S = { heads + heads, heads + tails, tails + heads, tails + tails }
  - $P \geq 1T = 3/4$
- die roll, S = {1,2,3,4,5,6}
- 3 baby = S = { ♀♀♀, ♀♀♂, ... }

Compound Events - 4.2

- $\boxed{P(A \text{ and } B) = P(A) \times P(B)}$ (independent)
- prob of 2 things happening at the same time
- Eg Dice - $P(5,5) = P(5) \cdot P(5)$

$\frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$ P(5,5)

0.02778 "highly unlikely"

Replace or Not

- Ace & Ace - Replacing!

$\frac{4}{52} \times \frac{4}{52} = \frac{16}{2704} \to 0.00592$

"highly unlikely"

- Ace & Ace - Don't Replace (has dependency!)

$\frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} \to 0.00452$

"highly unlikely"

↳ presuming you got ace on 1st try

Independence vs Dependence

- $\boxed{P(A,B) = P(A) \cdot P(B|A)}$ (Dependent)

↳ B depends on A must happen, conditions must be met

2 events are independent if $P(A) = P(A|B)$

4 Seasons

door 1 door 2

$P(UGLY) = \frac{1}{3} \cdot \frac{1}{2} = \boxed{\frac{1}{6}}$ 16.67% "regularly unlikely"

3 keys, 2 doors, which key

assuming success on door 1

$P(\$\$\$\$\$) =$

$\frac{5}{25} \cdot \frac{4}{24} \cdot \frac{3}{23} \cdot \frac{2}{22} \cdot \frac{1}{21} = 1.88 \times 10^{-5}$

$\boxed{P(A \text{ or } B) / P(A \cup B)}$ union

↳ either/or! as long as it happens, doesn't matter how.

$= P(A) + P(B)$ ★ adding!

Eg: $P(\text{King} \cup \text{Ace}) = P(K) + P(A)$

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

$\frac{4}{26}$ $\boxed{\frac{2}{13}}$ "pretty unlikely"

$P(K \cup \diamondsuit) = P(K) + P(\diamondsuit)$

$\frac{4}{52} + \frac{13}{52} = \frac{17}{52}$

$\frac{17}{52} - \frac{52}{2704} =$ wtu

★ Non mutually exclusive / non disjoint

① add all
② take away overlap at the end, so...

$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$

★ Summary ★

P(A and B) = **And**

P(A) • P(B|A)

P(A) • P(B) if independent

Or

P(A or B) =

P(A) + P(B) - P(A and B)

P(A) + P(B) if disjoint

- <u>Two-way Contingency tables</u>, used for conditionals (rows first then columns 3 by 2 1) 3 rows 2 columns)
  - used for example, P(positive test | condition present)

    look @ column for condition present first and then + test.

    the fraction is $\frac{\text{pos test}}{\text{con present}}$. First thing goes on numerator, second thing denominator
  - if <u>and</u> (eg P(positive test • condition present)) would go out of total number (for all) and the one that satisfies both conditions $\frac{\text{pos test and condition}}{\text{total \#}}$

CE

- P(A and B)
  - Ind: P(A)·P(B)
  - Dep: P(A)·P(B|A)
- P(A or B)
  - Disjoint: P(A)+P(B)
  - Nondisjoint: P(A)+P(B) - P(A and B)

4.3

Combinations:

- order doesn't matter
- $\frac{n!}{r!(n-r)!}$
- outcomes consist of nonordered subgroups of r items out of group of n items

Permutations: $(P_{n,r})$

- order matters
- $\frac{n!}{(n-r)!}$
- always bigger (unless r = 0 or 1, combination will be the same)
- ordered subgroups of r items from group of n items

Tree Diagrams

- Visual display of all the outcomes & shows individual outcomes
- multiplication rule of counting: $n_1 \times n_2 \times n_m \ldots$
  - n_1 is # of possible outcomes for event E_1, n_2 # of possible outcomes of event E_2, n_m is # of possible outcomes for event E_m.
  - gives total number of outcomes
  - eg 2 psych class, 2 physiology, 3 spanish = $2 \times 2 \times 3 = 12$ outcomes

Factorial

- # of order arrangements possible of the n items
- eg $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $0! = 1$

$P(+\text{ test}) = P(+\text{ test} \mid +\text{ man}) + P(+\text{ test} \mid -\text{ man})$

$= (.95)(.02) + (.98)(.12)$

$= 0.1366$

$P(+\text{ man} \mid +\text{ test})$

4.3 Study Guide

(2) When the outcomes are equally likely, how do we find P(A)?

$$\frac{\#\text{ of outcomes favorable to event A}}{\#\text{ of outcomes in the Sample Space}} = P(A)$$

(3) Identify the limitation to the approach in #2

- need to list sample space + number, but often gets too complex & large

(4) A visual display of outcomes made of a series of activities is:

- tree diagram

(5) Complications in tree diagram of course options?

- Some have jobs, which conflicts time wise, pre-req conflict, sleep conflict

(6) Tennis match tree diagram vs its sample space

- tree diagrams are a visual display of the sample space

(7) Calculate eleven factorial

$11! = 11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 39916800$

(8) When does 0=1?

$0! = 1! \rightarrow 0 = 1$

(9) The order of items is not important in combinations

-True

(10) Combinations: $\frac{n!}{r!(n-r)!}$; Permutations: $\frac{n!}{(n-r)!}$ } exception: they are the same if r is 0, or 1 (=1) — Permutations will be bigger always.

(11) In ex 13, why do they compute # of combinations?

nonordered, can be 4 books in any order

(12) What is meaning of $C_{n,r}$?

n= trials, r = # of successes

(13) Calculate $C_{10,4}$

"# of combinatons of 4 successes out of 10 trials" = $\frac{10!}{4!(6)!}$ = $\boxed{210}$

Calculate $P_{10,4}$

$\frac{10!}{(6)!} = \boxed{5040}$ much larger

(15) What does $P_{5,5}$ mean? $= \boxed{120}$

What does $C_{5,5}$ mean? $= \boxed{1}$

(16) math → prob → nPR, nCr

Pig

1) Probability of throwing one $1/6 + 1/6 = 2/6$ $\boxed{1/3}$

2) Probability of snake eyes: $1/6 \cdot 1/6 = \boxed{1/36}$

3) What is a lucky streak? Do you know one when you have it?

Quantitative asssesment of luck:

times you'd see one

$$\frac{10}{36} + \frac{1}{36} = \boxed{\frac{11}{36} = \frac{1}{R}}$$ how many rolls would it take to see a one

R = 3.27

ever 3.27 rolls, you should see a one

4 rolls is beyond that, so <u>4+ is lucky</u>

5.1 - Random Variables (X)

$\boxed{\text{Discrete}}$ RV: Quantitative, countable, integers, no fractions

- eg: # of students who voted in an election (can't be decimal)

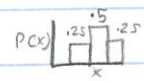
vs

$\boxed{\text{Continuous}}$ RV: quantitative, countless, infinite, -2, -1, 0, 1, 2, and fractions / decimals / partial values in between

- eg: temp in room, time the lesson takes (can be decimals)

Probability Distributions:

- assigns probabilites to each random variable
- usually a histogram (w/ discrete data)



- random variable on x-axis
- probability (P(x)) on y-axis
- all area adds up to 1 (bc no gaps / overlaps)

Center & Spread

→ expected value (mean)

$\boxed{\mu = \Sigma(x \cdot p(x))}$ ★

→ Standard deviation

$\boxed{\sigma = SD(x) = \sqrt{\Sigma (x-\mu)^2 \cdot p(x)}}$

$\Big\}$ Discrete

on calc ↑ insert in list L1 (x) + L2 (p(x))

1 var stats L1, Freqlist L2

Binomials

- Jacob Bernoulli: 17th century Swiss mathematician, esp binomials
- The sort of problems which have exactly 2 possible outcomes ($S = \{Y, N\}$) is called binomial (success & fail)
- Describe the central problem of a binomial exprmt: Probability of r successes out of n trials ($\frac{r}{n}$)
- Binomials only work with independent situations (cannot be dependent) = A major Assumption
- Each faculty member is asked about recommending which car model should purchase. 500 members.
    - # of trials, n = 500
    - # of outcomes possible = unclear, however many car models exist.
    - is this binomial experiment? No

Discrete RV

- integers, whole #s, add to 1, ha μ, σ, and make histograms
- binomial is part of it

binomials are really just percents

1) Success / Fail only options, which are mutually exclusive
2) Independent
3) Predetermined number of trials (n)
4) Probability stays the same for each trial ($\overset{\text{success}}{p} + \overset{\text{fail}}{q} = 1$, thus $q = 1-p$) determined by goals & research
5) ctrl goal: prob r/n (r successes out of n trials)

Example 1: At hospital, the staff is large to set a goal: 80% of the time a nurse will respond to a room call within 3 minutes. There were 72 room calls yesterday. We wish to find the probability nurses responded to 63 of them (7 out of every 8,) within the 3 minute goal.

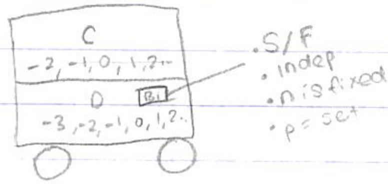
- Trial = a room call
- Independent? Yes
- S = a nurse got to the room in under 3 min
- F = a nurse got to the room in over 3 min
- P = .80 (80%)
- Q = .20 (20%)
- n = 72
- r = 63

Formula for binomial probability distribution

$-P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r}$

$P(r) = C_{n,r}\, p^r q^{n-r}$

<u>notation</u>: $(P_{r=5}) = 0.01202$



* expected value (μ): $n \cdot p$ (only for binomials, usually $\Sigma x \cdot p(x)$)

* Standard deviations: $\sigma = \sqrt{np(1-p)}$

$(\sqrt{npq})$

$(\sqrt{\mu q})$

5.4 Geometric Distribution

Hurrican example: 41% ts become h.

2 — S = TS → H; F = TS

P that 4th ts will be 1st hurricane?

H = 74 + mph

Independent

$p = .41$
$q = .59$

H = the # of ts it takes to get the first hurricane

fail ... success

$P(H=4) = (1-0.41)(1-0.41)(1-0.41)(0.41)$

$= 0.0842$

basically $P(H=4) = q \cdot q \cdot q \cdot p$

- 2 possible outcomes
- Independent
- Each trial has same prob

also in binomial

- Does not have a fixed number of trials (non)
- $P(X=n) = (1-p)^{n-1} p$ for $x = 1, 2, 3$
- $\boxed{P(X=n) = q^{n-1} \cdot p}$

Mean Number: $\boxed{\mu_x = \frac{1}{p}}$

Standard deviation: $\boxed{\sigma_x = \frac{\sqrt{q}}{p}}$

$\boxed{P(x > n) = q^n}$

Ex: 20% of animals have 4+ pups.

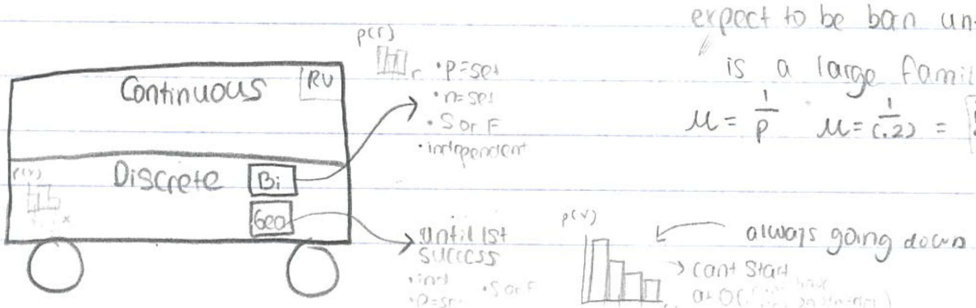
(a) P of waiting til 5th litter is born to have 4+?

$P(x=4+) = (.8)^4 \times (.2) = \boxed{0.0812}$

(b) How many litters should expect to be born until there is a large family?

$\mu = \frac{1}{p} \quad \mu = \frac{1}{(.2)} = \boxed{5 \text{ litters}}$

all are histograms



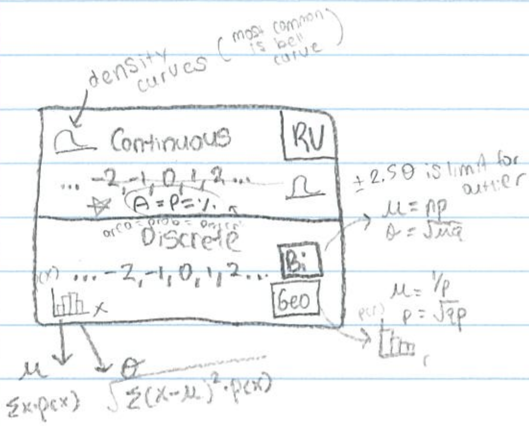
1/10/24

Stats Rvw

dice	x	p(x)	x(p(x))
1	\$8	1/6	8/6
2	\$-2	1/6	-2/6
3	\$-2	1/6	-2/6
4	\$-2	1/6	-2/6
5	\$-2	1/6	-2/6
6	\$-2	1/6	-2/6

$\mu = \Sigma = -2/6 = \boxed{\$-1/3}$

negative expected earnings ☹

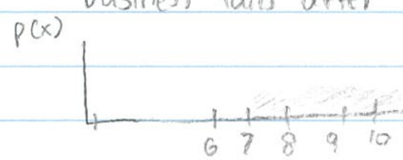


★ 4 things for credit!

1) Notation
2) Answer
3) ∃x a ... % chance that ...
4) Graph / Sketch

eg: $p(X>6) = .21$

∃X a 21% chance that the business fails after 6 yrs



Insurance

Loss or no loss

nL: \$625

L: \$-200,000+650-25 = -199,375

x	p(x)	x·p(x)
\$625	.998	623.75
\$-199,375	.002	-398.75

$\mu = \Sigma = \$225$