8.1 Hypothesis Tests

Hypothesis: assumption/belief about a parameter

↳ Testing: procedure, based on sample info, by which one "accepts" (fail to reject) or "rejects" hypothesis

① Null hypothesis: $\boxed{H_0}$ – hypothesis set up to test whether or not can be rejected. "No change/difference" tradition

② Alternate hypothesis: $\boxed{H_1}$ or $\boxed{H_a}$ – hypothesis to be accepted if H_0 is rejected

Testing Errors

1) Type I: reject the null even though null is true

- α = "lvl of significance"

$\boxed{\alpha = 1 - C}$

P of flubbing

2) Type II: accept the null even though its false

- $\boxed{B}$ P of flubbing

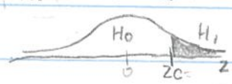
Power of a test

- $\boxed{1-B}$
- rejecting null when it is false
- increasing n increases power

4 Ingredients of Statistical Test

① $H_0: \pi = 93\%$

② $H_a: \pi > 93\%$



③ critical value (separates Ho from critical region)

④ sample statistic: $\hat{p} = .939 = Z = .123$ / point estimate $(\bar{x}, \hat{p})$

↳ convert it into a Z_c : $\boxed{Z = \frac{SS - PP}{SE}}$ $\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right)$

SE → standard error

SS → sample stat

PP → pop'n parameter

✓ optional

⑤ p-value

8.2 Part 1: Z when σ known

include $\sqrt{s}$

"5 = 1" & "1 = 2"

using $\alpha = .05$, the cr will be 1.645 (1-tail) or 1.96 (2-tail)

using $\alpha = .01$, the cr will be 2.33 (1-tail) or 2.58 (2-tail)

note: "previous studies" means popln data

rare = 5% or less

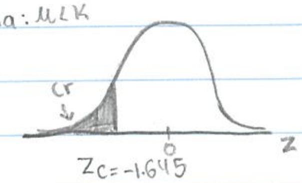
"level of significance": $\alpha = .05$ ← go-to if not stated in question

↳ probability of Type I error

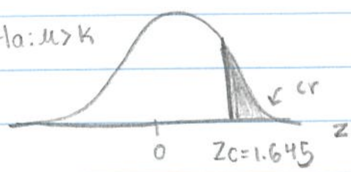
willing to risk messing up 5 times in 100

$\alpha = 0.05$

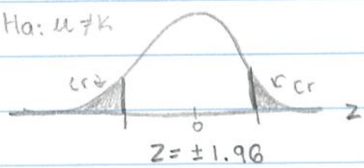
Ha: $\mu < K$



Ha: $\mu > K$

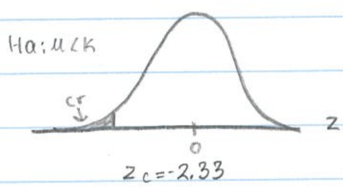


Ha: $\mu \neq K$

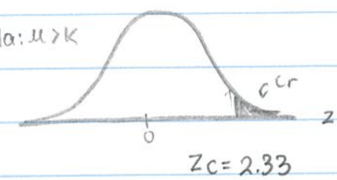


$\alpha = 0.01$

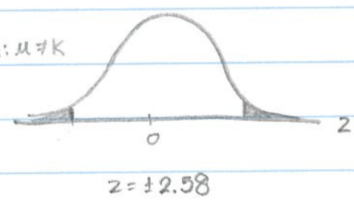
Ha: $\mu < K$



Ha: $\mu > K$



Ha: $\mu \neq K$

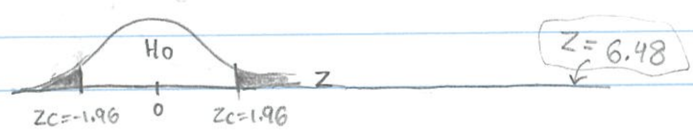


Ex: $\mu = 130$, $\bar{x} = 131.08$, $n = 81$, $\sigma = 1.5$, $\alpha = .05$, "a contradctn" → $\neq$

Ho: $\mu = 130°F$ (units!) The popln mean for this company is 130°F.

Ha: $\mu \neq 130°F$

$\bar{x} \to Z$: $\frac{131.08 - 130}{1.5/\sqrt{81}}$



$\boxed{Z = 6.48}$

Conclusion:

- Reject the null, accept the alternate, alpha is .05
- 3x sufficient statistical evidence suggesting this company's sprinkler system activates at a temp different than 130°F (3x sses)
- p-lvl ≈ 0

8.2 when σ unknown

$\sigma \to$ ⌒ t

df used

$\bar{x}$ – $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

- Calculate t-value, and see where it falls on t-table using proper df (youre looking for area, based on 1/2 tail)
- it will give you a range of P-values
  - if 2-tail, dont have to x by 2 (like we did w/ P-values when σ known, since already gave 2-tail
- Smaller P value = reject! (ex, $0.02 < P\text{value} < 0.05$ > $\alpha = 0.01$

ex: two tail, t = 2.108

	...	...
one-tail / ✓ two-tail	0.050	0.020
df = 20	2.086	2.528

t falls here

- t_c is found using df and lvl of significance! "given"

(so if 0.01, and two-tail, use that for top)

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$\boxed{8.2}$ Part 2

p-value: probability of getting a value that "extreme"

- when 2-tail test, multiply by 2
- small P-values give evidence against Ho.
  " if the p value is low, then the Ho has to go! "
- Compare p with lvl of significane
  eg p level = .2802 > $\alpha = 0.05$
  "weun" weak evidence against the null
  "Seun" strong evidene against the null
- the smallest significance level at which we reject Ho.
- p of getting the result if the null is true

use z-table!

8.3

- %s
- $\hat{p} = r/n$
  - ↑ Binomial $<^{S}_{F}$
- Z

$$\boxed{Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}}$$

Note: small zs = big Pvalues
+ big zs = small P values

Checks (% intervals/tests)

- RRS
- Indep
  - $n \le N/10$ ← rsnbl...
- $\left.\begin{array}{l} np \\ \text{and} \\ nq \end{array}\right\rangle \ge 10$ "ANAIJ"
  - → z

make it flow in order!
(normal → z/t)

Z- Interval/Test Checks

- RRS
- Independent
  - $n \le \frac{N}{10}$ ~> it is reasonable to assume Ex $10(n)$ + ____ to study/sample our n from
- "normally distributed"

  or

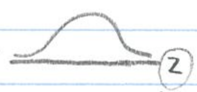
  "mound/symmetric"

  or

  $n \ge 30$ ... CLT invoked

  or

  pearsons index, $-1 < pi < 1$ or NQP or histogram



- σ known

T- Interval/Test Checks

- RRS
- Independent
  - $n \le \frac{N}{10}$
- "normally distributed"

  or

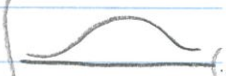
  "mound/symmetric"

  or

  $n \ge 30$ ... CLT invoked

  or

  pearsons index, $-1 < pi < 1$ or NQP or histogram



- σ unknown, S known

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

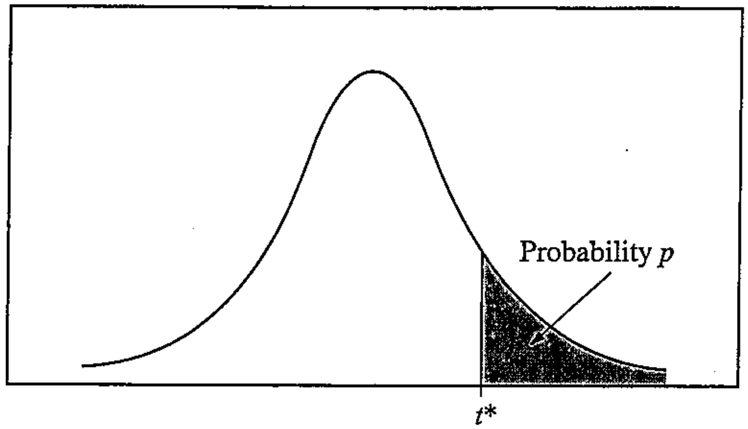


Table B *t* **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

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2/12/24

Math 210 Intro to Prob and Statistics

1. Error of depth perception is very important in dental work. A certain aptitude test asks subjects to estimate the distances between a fixed object and a second object of variable position. A random sample of 14 subjects gave the following information about errors in a particular depth perception test (units in millimeters):

1.1	1.5	0.9	2.1	1.4	1.7	0.8
1.3	1.8	1.1	1.6	1.9	1.2	1.6

a. Use a calculator to verify that the sample mean of the above data is 1.43 mm and the standard deviation is 0.38 mm. $\bar{x}=1.43$ $s=0.3832$

b. Find 90% confidence interval for the population mean of errors for this depth perception test.

$df=13$ $(n=14-1)$

$c.l.=90\%$

$t_c = t_{.90} = 1.771$

$E = 1.771\left(\frac{0.38}{\sqrt{14}}\right)$

$E=0.1799$

$\boxed{1.2501 < \mu < 1.6099 \text{ millimeters}}$

If we took 100 samples of size n=14, we expect to capture the popln mean (μ) millimeters distance 90 ocasions.

Checks
1. RRS
2. Independent

2. Shoplifting has been a problem in a large men's clothing store. Using special security measures to monitor shoplifting, it was found that there were attempts to shoplift the following dollar values of merchandise each week for the past nine weeks:

$356	$285	$310	$375	$290
$325	$331	$342	$335	

a. Use a calculator to verify that the sample mean is $327.67 with sample standard deviation $29.31.

b. Find a 90% confidence interval for the population mean.

label: Z-test for 2 independent means

σ known

8.5 Part 1: Diffance of Two Means: Independent

- Not dependent (like 8.4), separate sampes, dont have to be related
- same conditional checks (+ mention $\sigma_1 + \sigma_2$ known, + $n < \frac{N}{10}$ for both samples)
  - can have different sample sizes
- will have σ_1 and σ_2 known (2 diff), but you will combine to get a grouped sigma
- Hypothesis statements
  - $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$
  - $H_a: \mu_1 - \mu_2 \lesseqgtr 0$ or $\mu_1 \lesseqgtr \mu_2$
- Z-value

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \rightarrow \boxed{z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

calc: Stat → test → 3 (2 samp z test)

- everything else same (find p, on z-table, pol z score, etc)
- label which one is μ_1 and μ_2 (sentence)
- add the variable you are measuring in concl (not just no diff)

σ unknown

Part 2: Difference of Two means: Independent

- also independent
- same H_0 & H_a as P1
- t-value:

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

1) df = $n_1 - 1$ or $n_2 - 1$, whichever is smaller
2) another way of getting df... $\boxed{n_1 + n_2 - 2}$ (which is $n_1 - 1 + n_2 - 2$)
3) or use calc (may get decimal)

} for getting t_c

Part 3:

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7.4

7.4 Part 1

CI of $\mu_1 - \mu_2$, σ_1 / σ_2 known (z-interval)

1) find $\bar{d}$, + z_c
2) $\mathcal{E} = z_c\left(\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

3) $\bar{d} - \mathcal{E} < \mu_1 - \mu_2 < \bar{d} + \mathcal{E}$

Part 2

on calc, 8

CI of $\mu_1 - \mu_2$, σ_1 / σ_2 unknown (t-interval)

1) find $\bar{d}$
2) df is smaller of either $n_1 - 1$ or $n_2 - 2$
3) find t_c
4) $\mathcal{E} = t_c\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$
5) $\bar{d} - \mathcal{E} < \mu_1 - \mu_2 < \bar{d} + \mathcal{E}$

if σ_1 is assumed to $\approx \sigma_2$ (rare), you can pool. use $df = n_1 + n_2 - 2$

$$\mathcal{E} = t_c \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}$$

pooled sd

Part 3

on calc - B

CI of $p_1 - p_2$ (z interval)

$$\mathcal{E} = z_c\left(\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$(\hat{p}_1 - \hat{p}_2) - \mathcal{E} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + \mathcal{E}$$

★

Conclusion, 2 parts

(1) If we took ___ samples all of about the same size, we expect to catch the popln difference of means ($\mu_1 - \mu_2$) ___ on ___ occassions.

(2)

Option 1:

$+ \text{---} + \rightarrow \mu_1 - \mu_2 > 0,$
$\mu_1 > \mu_2$

positive to positive interval implies that μ_1 is greater than μ_2, thus ___

Option 2:

$- \text{---} - \rightarrow \mu_1 - \mu_2 < 0$
$\mu_1 < \mu_2$

negative to negative interval implies that μ_1 is less than μ_2, thus ___

Option 3:

$- \text{---} + \rightarrow \mu_1 \approx \mu_2$

negative to positive implies that μ_1 and μ_2 are about the same, thus ___

$\boxed{8.4}$ Hypothesis Tests: Dependent

- **Before/After**: looking at difference of 2 groups
- **Matching Link**: natural matches/pairs (feet)
  ↳ matched data reduces the danger of extra factors except for the one we wish to measure, reduces variance

$\boxed{\bar{d}}$ → d bar = mean difference ($\bar{x}$) between matched / paired data
- basically sample mean, is for the diff

$\boxed{S_d}$ → S sub d is the sample standard deviation ($\approx S_x$)

$\boxed{\begin{array}{l} H_0: \mu_d = 0 \\ H_a: \mu_d \gtrless\neq 0 \end{array}}$ (no difference)

On calc:
- input $L_1 + L_2$ (p1, p2), L_3 is $L_1 - L_2$ (diff between a pair)
- 1-var-stats for L_3 to get $\bar{x}$ + S_x ($\bar{d}$, S_d)

Checks:
- same as t-tests except for "indep"
- it is both ind + dep (pairs dep, but sep pairs are indep)

t-tests:
- robust: works for non-normal distributions
- small samples ok

Name:
T-test of dep mean