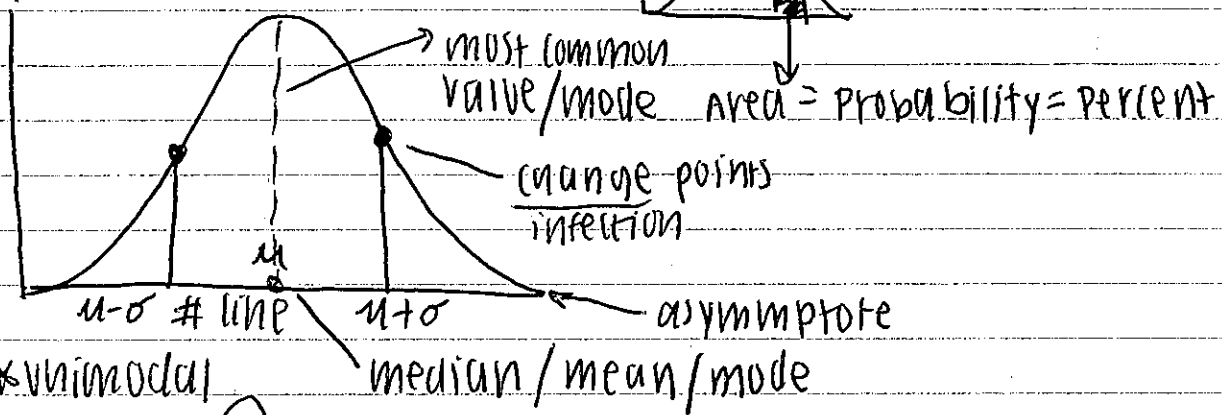
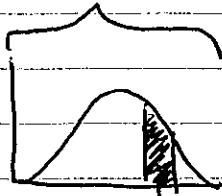
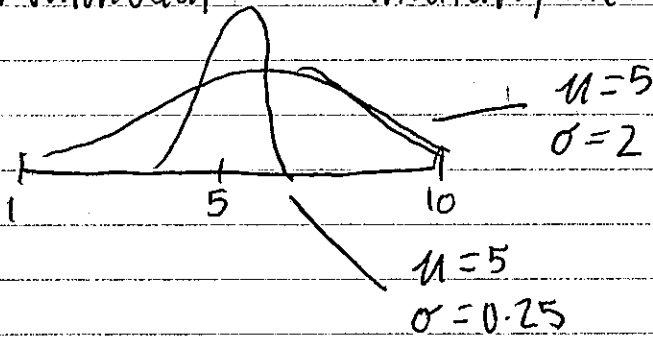


# CONTINUOUS VARIABLES AND CURVES 1/12/24

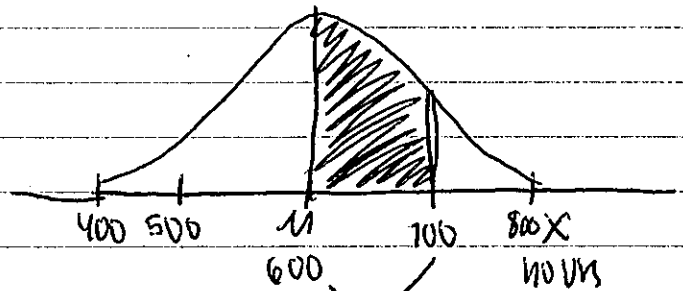
CONTINUOUS CURVES  
 density curves, bell curves



\*unimodal



SUNSHINE RADIO EXAMPLE:



SD = 100

34%

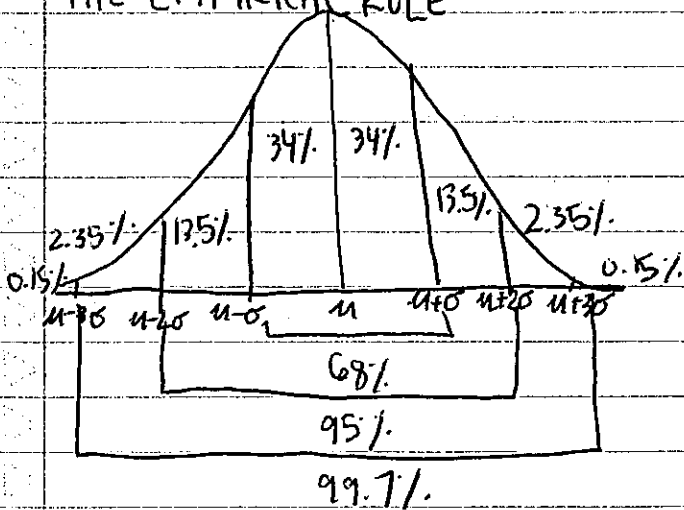
MEAN = 600

$P(600 < x < 700) = 0.34$

$\exists$  a 34% chance that SR lasts 600-700 hours

$<$  and  $\leq$  makes NO DIFFERENCE

## THE EMPIRICAL RULE



# Z SCORES AND Z CURVES 1/17/24

BIO

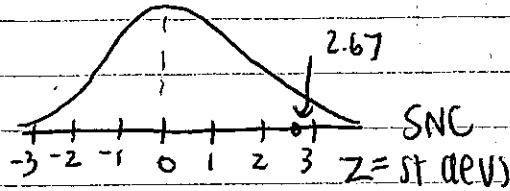
$\mu = 70$

$X = 74$

$\sigma = 1.5$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z_{bio} = \frac{74 - 70}{1.5} = 2.67$$



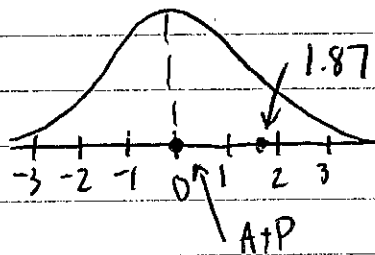
HIST.

$\mu = 86$

$X = 91$

$\sigma = 2.67$

$$Z_{hist} = \frac{91 - 86}{2.67} = 1.87$$



ANAT + PHIS

EX: ~~PIZZA PROBLEM~~ #9: FIRST AID COURSE

ROBERT: 1.10

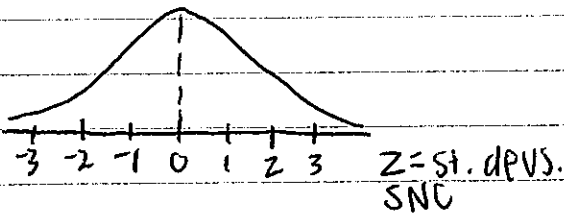
JUAN: 1.70

SUSAN: -2.00

JOEL: 0.00

JAN: -0.80

LINDA: 1.60



a. above mean ~ ROBERT, JUAN, LINDA

b. ON THE MEAN ~ JOEL

d.  $\mu = 150$   $\sigma = 20$

$$Z = \frac{X - \mu}{\sigma}$$

ROBERT:

$$1.1 = \frac{X - 150}{20}$$

$$\boxed{172 = X}$$

SUSAN:

$$-2 = \frac{X - 150}{20}$$

$$\boxed{X = 110}$$

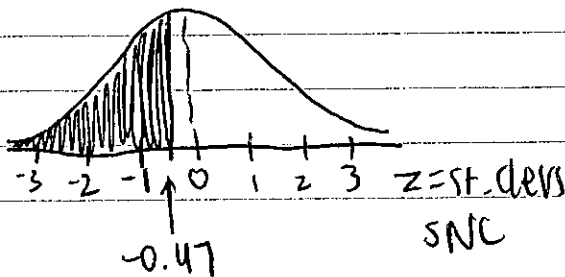
$$22 = X - 150$$

$$-40 = X - 150$$

HW

16.

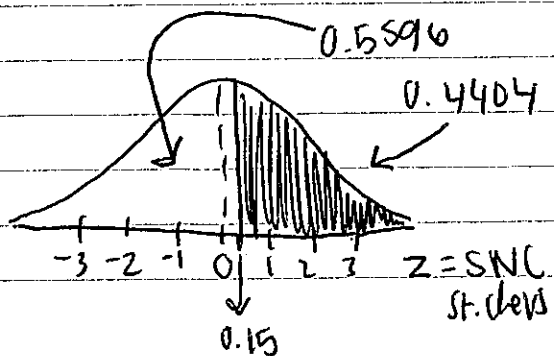
0.3192



20.

0.5596

0.4404

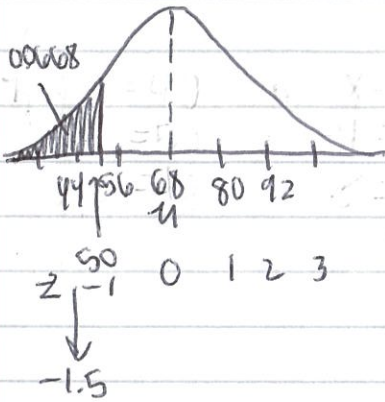


REVIEW:

Pacific Yellowfin Tuna

$\mu = 68$   $\sigma = 12$  (lb)

$$z = \frac{x - \mu}{\sigma}$$

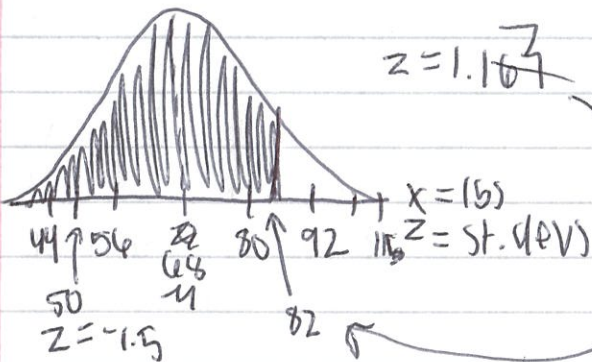


$$P(X < 50) = 0.0668 = \frac{50 - 68}{12} = -1.5$$

$\exists$  a 6.68% chance

that a PYT (caught is 1P) than 50 lbs

$x = 105$   
 $z = \text{st. dev}$



$$\frac{82 - 68}{12} = 1.167$$

$$P(50 < X < 82) = 0.8122$$

$$0.8790 - 0.0668 = 0.8122$$

$\exists$  a 81.22% that a PYT (caught is b/w 50 and 82 lbs

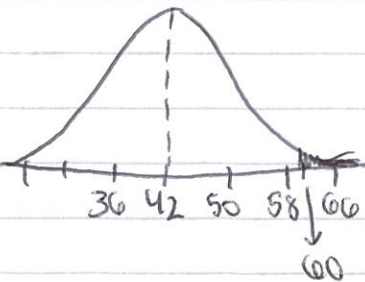
CALC WAY:

2nd  $\rightarrow$  VARS

2: normalcdf

Flight for life

$\mu = 42$   $\sigma = 8$  longer than 60



$$P(X > 60) = 0.0122$$

$\exists$  a 0.122% chance that ~~the~~ flight for life helicopter with take more than 60 min to return

$x = \text{min}$

## COLLEGE BOARD VIDEO NOTES (1/20/2024 @ 37)

### 4.2 ESTIMATING PROBABILITIES USING SIMULATION

**LAW OF LARGE NUMBERS** - simulated probabilities seem to get closer to the true probability as the # of trials increases

↳ variability from true probability also decreases

### CONDUCTING A SIMULATION

Random # generator

- describe how the digits imitate the trial
- ~~after~~ determine what is recorded from each trial
- perform many trials
- calculate relative frequency of successful trials

→ Example: sharpshooter

#1-82 = made

OR

#01-82 = made

#83-100 = missed

#83-99,00 = missed

### 4.9 COMBINING RANDOM VARIABLES

**PROBABILITY DISTRIBUTION** num didn't take away is e thing

~~~~~

4.9 pt 2. num doesn't make sense

Binomial:

-  $n = \text{trials}$

$$\mu = np$$

- S/F

$$\sigma = \sqrt{npq}$$

- independent

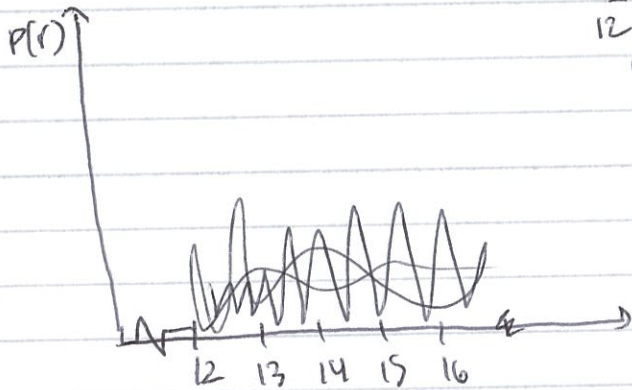
- probability is given  $p = q =$

-  $r$  successes  
in  $n$  trials

#9 drug dealers

$$P(X \geq 12) = 0.028 + 0.009 + 0.002 = 0.039$$

Ex a 7.9% (maybe that  
12/16 or more ...)



~~binomcdf~~

binomcdf

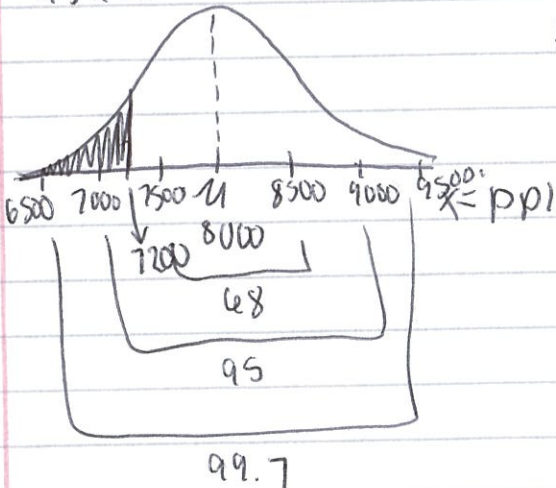
trials: 16

$p = 0.5$

$x = 11$

$1 -$   $\rightarrow$   $=$   $\text{☺}$

#36a.



$$P(X < 7200) = 0.0548$$

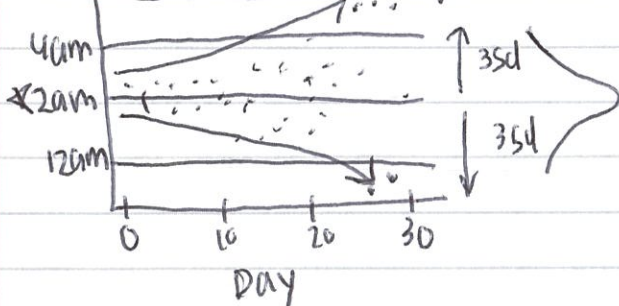
normalcdf

Ex a 5.48% ...

## Against All Odds Notes 1/25

- W. Edward Deming ~ statistical ~~process~~ <sup>product process</sup> control SPC
  - "Japanese Miracle"
- process ~ chain of steps that turns inputs into outputs
- control charts used to monitor control
- ~~variation~~ common cause variation ~ normal human errors
- special cause variation ~ sudden and unexpected

specimen processing finish time - Month 1



- more inconsistent
- less reliable

- points out of the barriers  
if out of control

### Decision Rules

- ① one point out of control
- ② 2 of 3 points b/w 2 and 3 $\sigma$ 's on the same side of mean
- ③ 4 of 5 points on same side of mean
- ④ 8 in a row on the same side of mean  
↳ book says 9

- random variable  $x$  in statistical control

### PROCEDURE:

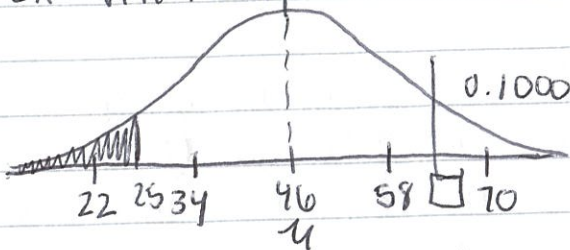
- ① find  $\mu$  and  $\sigma$  of  $x$  distribution
- ② create graph vertical axis =  $x$  horizontal = time
- ③ control limits  $\mu \pm 2\sigma$  and  $\mu \pm 3\sigma$
- ④ plot

### Warning signals:

- ① Beyond  $3\sigma$
- ② 9 consecutive all below / above center  $\mu$
- ③ at least 2 of 3 consecutive points beyond  $2\sigma$

# 7.3 NORMAL? 1/25/2024

EX: Veterinary Science  $\mu = 46$  beats/min 95% data 22-70



$\sigma = 12$   
 $70 - 22 = \frac{48}{4}$

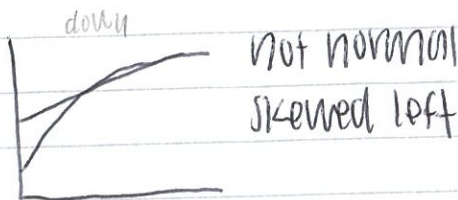
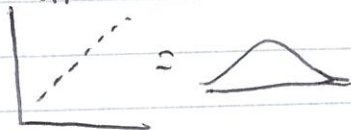
b.  $P(X < 25) = 0.0375$  400 399 normcdf(1, 25, 46, 12)  
 e. upper 10%: invnorm(0.1, 46, 12) = 61.38 bpm

## CHECKS FOR NORMALCY:

1. Histogram  $\sim$  roughly bell shaped
2. Outliers
  - no more than 1
  - box-and-whisker to check
  - above  $Q_3 + 1.5 \times IQR$
  - below  $Q_1 - 1.5 \times IQR$

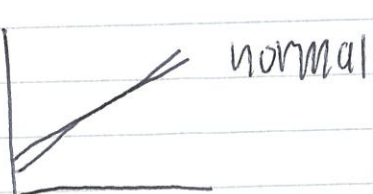
3. Skewness  $\sim$  normal are symmetric  $\checkmark$   
 Pearson's Index =  $\frac{3(\bar{x} - \text{median})}{sd}$   $-1 \leq PI \leq 1 = \text{normal}$

## 4. Normal quantile plot (N probability P)



CALC:

2nd  $y =$  stat plot



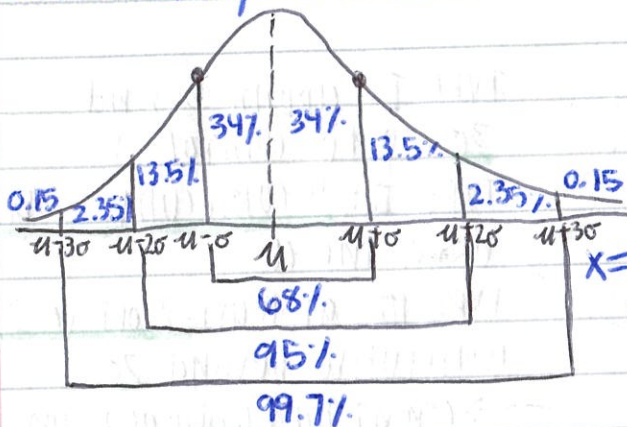
nick fails

# UNIT 7:

(CHAPTER 6):

NORMAL CURVE: EMPIRICAL RULE, Z SCORES, AREA = PROB

## CONTINUOUS / NORMAL CURVE:



- bell-shaped, unimodal
- symmetric over  $\mu$  (mean, med., mode)
- never touches line  $\sim$  asymptote
- inflection points

$x = \star$   
need!

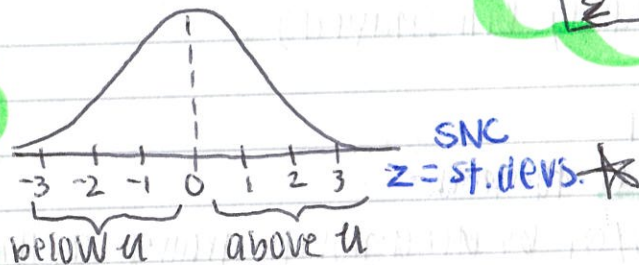
4 THINGS:

- NOTATION
- ANSWER
- Ex a....

$<$  and  $\leq$   
same

## Z-SCORES:

$$Z = \frac{x - \mu}{\sigma}$$



## → Z TABLE (TABLE 5)

area left = -  
area right = +

## → CALL WAY

2nd → vars  
z: normal cdf

upper  
lower/ ~~area~~ =  $4\sigma \pm \mu$

← cumulative

## INVERSE NORMAL:

% is given

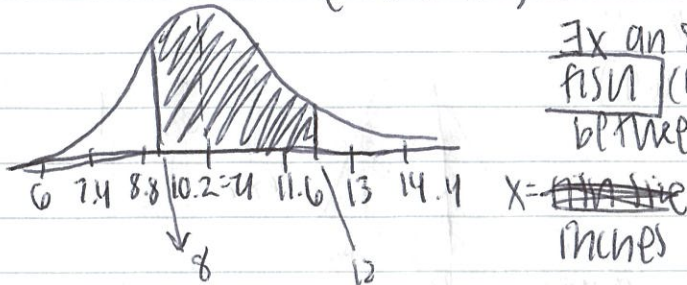
2nd → vars

3: invNorm( % as decimal,  $\mu=0$ ,  $\sigma=1$ , tail: CENTER)



REVIEW

CHILDREN'S FISHING POND SNC  
 $\mu = 10.2''$   $\sigma = 1.4''$   $P(8 < X < 12) = 0.8426$

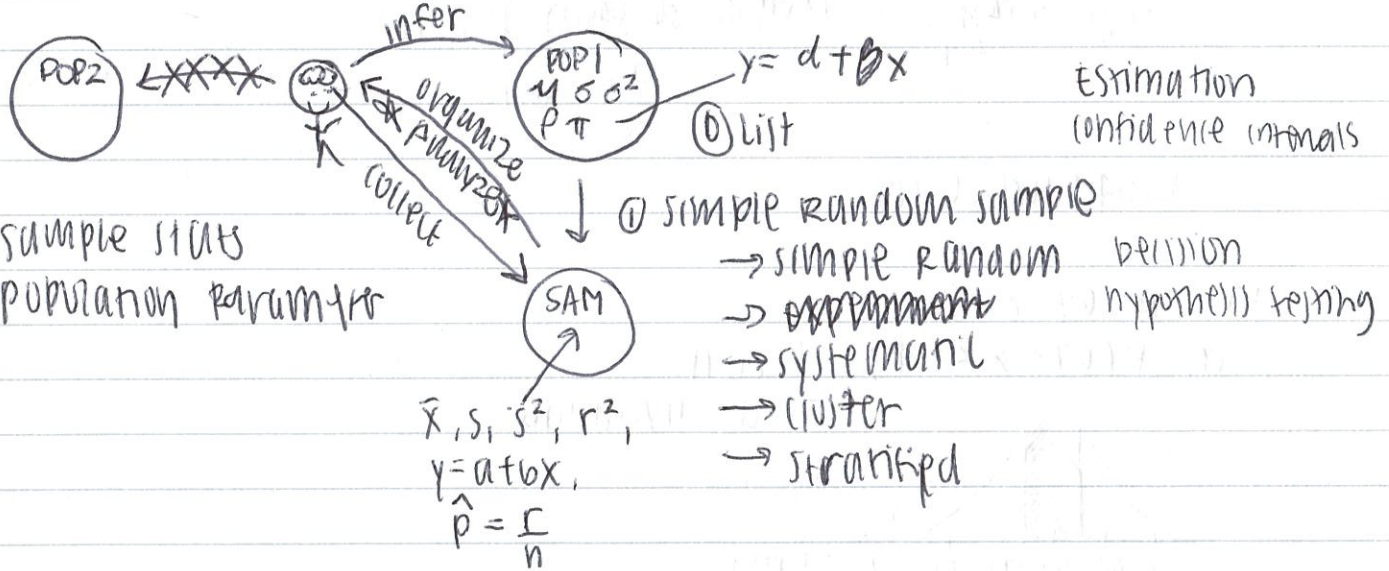


Ex an 84.27% chance that a random fish caught at children's fishing pond is between 8" and 12"

6.4

population = all measurements of interest

sample = a subset of the measurements from the pop



sample stats

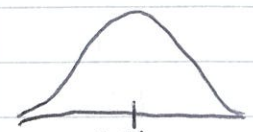
population parameters

$\bar{x}, s, s^2, r^2,$   
 $y = a + bx,$   
 $\hat{p} = \frac{c}{n}$

- ① simple random sample
- simple random
- experiment
- systematic
- cluster
- stratified

THEOREM 6.1 ORIG X DISTR

NORMAL



same  $\leftarrow \frac{\mu}{\mu}$   
 diff  $\leftarrow \frac{\sigma}{\sigma\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$\bar{x} = \text{avg of } 5 \sigma$

$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

NOT NORMAL



$\frac{\sigma}{\sqrt{n}}$  = standard error

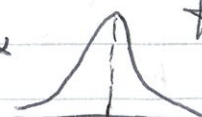
THEOREM 6.2  $\bar{x} =$



$\sigma\bar{x} = \frac{1.4}{\sqrt{5}} = 0.6261$

hammer  $\sigma$  shrinks when avg sizes

$\star n = \text{BIG } \geq 30$



$\bar{x} = \text{avg}$

$\star n$  has to start out normal

$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

50 PENNIES AGES 1/31/24

| P # | BYr  | Age    |
|-----|------|--------|
| 1   | 2014 | 10 ✓   |
| ②   | 2023 | 1 ✓ ☆  |
| 3   | 2021 | 3 ✓    |
| ④   | 2010 | 14 ✓   |
| 5   | 2003 | 21 ✓   |
| ⑥   | 1968 | 56 ✓   |
| 7   | 1999 | 25 ✓   |
| 8   | 1998 | 26 ✓   |
| ⑨   | 1984 | 40 ✓   |
| 10  | 2022 | 2 ✓    |
| ⑪   | 2019 | 5 ✓    |
| 12  | 2003 | 21 ✓   |
| ⑬   | 1981 | 43 ✓   |
| 14  | 1964 | 60 ✓ ☆ |
| 15  | 2005 | 19 ✓   |
| 16  | 1998 | 26 ✓   |
| 17  | 1981 | 43 ✓   |
| 18  | 1998 | 26 ✓   |
| ⑱   | 1981 | 43 ✓   |
| 20  | 1983 | 41 ✓   |
| 21  | 1996 | 28 ✓   |
| 22  | 2001 | 23 ✓   |
| ⑳   | 2014 | 10 ✓   |
| ㉑   | 2019 | 5 ✓    |
| ㉒   | 2001 | 24 ✓   |

Math → prob randInt mean:  $24.1 \approx 24$

lower: 1

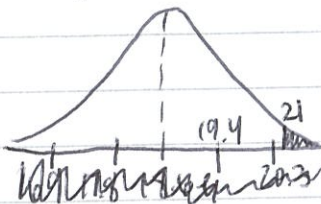
upper: 25

n: 10

## 6.5 CENTRAL LIMIT THEOREM

ACT PROBLEM  $\mu = 18.6$   $\sigma = 5.9$

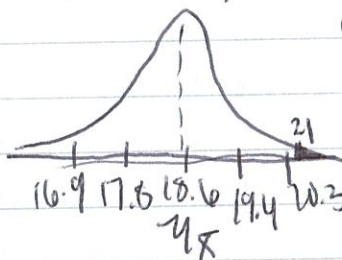
$$P(X \geq 21) = 0.2409$$



MILK did it

$X = \text{ACT score}$

$$P(\bar{X} \geq 21) = 0.0020 \quad n = 50 > 30 \therefore \text{CLT invoked}$$



$$\sigma_{\bar{X}} = 5.9 / \sqrt{50} = 0.8344 \quad \rightarrow \text{avg score on ACT}$$

Ex a 0.2% chance that a random group of 50 students is more than 21 ~~more than 21~~

$\bar{X} = \text{avg ACT score for 50 students}$

1,000 bets  $\mu = -0.0530$

$$\sigma = 0.998$$



100,000 bets  $\mu = -0.0530$

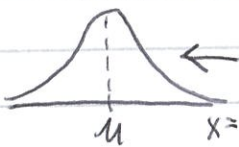
$$\sigma = 0.03$$



# CENTRAL LIMIT THEOREM

## ORIGINAL $x$ DISTRIBUTION

NORMAL

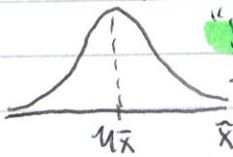


\* HAS TO START NORMAL;  
CAN WORK WITH big/small  
 $n$ 's

$$\mu = \mu_{\bar{x}}$$

$$\sigma \neq \sigma_{\bar{x}}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



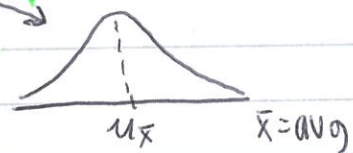
"standard error"

$\bar{x}$  is narrower/tighter

NOT NORMAL



doesn't matter



\*  $n = \text{BIG} \geq 30$   
 $\therefore$  CLT IS INVOKED

→ CALC WAY ~ 2nd → VARS

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

normalcdf ( —, —,  $\sigma/\sqrt{n}$  )

## 6.6 Normal Appx to Binomial 2/6/24 (Smart+svbash)

vaccine ~ binomial  $p=0.85$  300 ppl proportion/percent

Normal approximation

$n$  = trials  $\rightarrow$  higher  $n$  = closer to normal distribution

$r$  = successes  $np > 5$  } 10 for exam

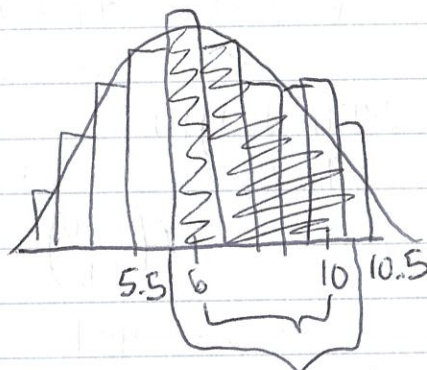
$p$  = success  $nq > 5$  }

$q = 1 - p$  failure

continuity correction - turning  $r$  into  $x$

- left point  $x = r - 0.5$

- right point  $x = r + 0.5$



EXAMPLE:  $n=40$  trial  $p=0.50$

$$np = (40)(0.5) = 20 > 5$$

$$nq = (40)(0.5) = 20 > 5$$

$\therefore$  a normal approximation is justified

ANAL

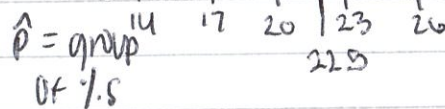
$$\mu = np \rightarrow 20$$

$$\sigma = \sqrt{npq} \rightarrow 3.16$$

$$r \geq 23 = x \geq 22.5 \quad P(x \geq 22.5) = 0.2146$$

CALC:

$z$ : normalcdf



EXAMPLE:  $n=25$  8 years 10 years 0.25

$$P(X \geq 7.5) = 0.2819$$

$$\hat{p} = r/n \quad \mu_{\hat{p}} = p \quad \text{and} \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

Still have to check