Name Key Group Date

Exploration 24: Parametric Function Graphs

Objective: Analyze the motion of an object whose path is given parametrically.

1. As Adam Ant crawls along the xy-plane, his position (x, y) is given by

   $x = 0.4t \cos t$
   $y = 0.3t + 2 \sin 2t.$

   Plot the path on your grapher. Use a square window of about –5 to 5 in the x-direction and –2 to 5 in the y-direction. Use a t-range of 0 to 4π. Have your instructor check your graph.

2. Find equations for dx/dt and dy/dt. Evaluate the two derivatives at $t = 6$.

$\frac{dx}{dt} = .4\cos t - .4t\sin t$ $\left.\frac{dx}{dt}\right|_{t=6} = 1.055$

$\frac{dy}{dt} = .3 + 4\cos 2t$ $\left.\frac{dy}{dt}\right|_{t=6} = 3.675$

3. Use the answers to Problem 2 in an appropriate way to find the slope of the path at $t = 6$. Offer an explanation, based on the graph, why your answer is reasonable.

$\frac{dy}{dx} = \frac{.3+4\cos 12}{.4\cos 6 - 2.4\sin 6} = 3.485$

Graph is increasing, positive slope.

4. Use the CALCULATE feature of your grapher to find the answer to Problem 3 numerically.

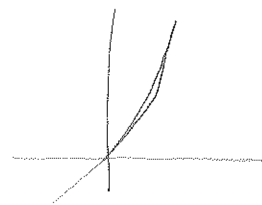
$\frac{dy}{dx} = 3.485$

5. Adam starts out at the origin and goes to the "north-east" for a while. At approximately what value of t does he turn around and start coming back?

$t = .83$

(I used TABLE to look for change in y)

6. Reset the t-range to go from 0 to the time Adam first arrives back at $y = 0$. Use a t-step of 0.01 to get a fairly accurate graph. Sketch the result here.



7. Find precisely by calculation the value of t at which y first stops increasing and starts decreasing.

$\frac{dy}{dt} = .3 + 4\cos 2t = 0$

$2t = \cos^{-1}(-.075)$

$t = .823$

8. At the time in Problem 7, is Adam stopped, or is he still moving? Justify your answer.

$\frac{dy}{dx} = 0$

$\frac{dx}{dt} = .0307$

speed $= .0307$ } moving horizontally

9. Use the ZOOM BOX feature to zoom in on the point in Problem 8. How does your answer confirm (or refute!) your conclusion in that problem?

$\frac{dy}{dx} = 0 \rightarrow$ Horizontal Tangent

Not a cusp

10. What did you learn as a result of doing this Exploration that you did not know before? (Over)

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Question 3

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.
  (i) The two values of t when that occurs
  (ii) The slopes of the lines tangent to the particle's path at that point
  (iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

Solution	Scoring
(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second	1 : answer
(b) $x'(t) = 2t - 4$ Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \ne 0$ This occurs at $t = 2.20794$. Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.	3 : $\begin{cases} 1 : \text{considers } \frac{dy}{dx} = 0 \\ 1 : t = 2.207 \text{ or } 2.208 \\ 1 : \text{direction of motion with reason} \end{cases}$
(d) $x(t) = 5$ at $t = 1$ and $t = 3$ At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right\|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=1} = 0.432$. At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right\|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=3} = 1$. $y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$	3 : $\begin{cases} 1 : t = 1 \text{ and } t = 3 \\ 1 : \text{slopes} \\ 1 : y\text{-coordinate} \end{cases}$